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30mm - 70mm WEAPONS STUDY

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FINAL REPORT

Contract DA-04-495-Ord 459

for

DEPARTMENT OF THE ARMY

Los Angeles Ordnance District · Pasadena California

1 February 1954

[illegible] COMPANY, AIRCRAFT DIVISION, C[illegible] CITY, CALIFORNIA

[illegible]

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TABLE OF CONTENTS

SUMMARY

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SUMMARY

This report covers the 30mm-70mm Weapon Feasibility Study conducted under Contract DA-04-495-Ord-459. As a result of the study it is shown that it is feasible to meet or exceed the kill probabilities required at the specified combat conditions with a 37mm gun based on the open-chamber principle.

The report is divided into seven sections as follows:

<u>Section 1</u> covers the description of the weapons principle that is proposed by this contractor as well as the historical data regarding a limited experimental firing program conducted to prove the feasibility of the approach.

<u>Section 2</u> considers certain other weapons principles as they apply to this study.

<u>Section 3</u> is concerned with the design considerations that demonstrate the feasibility of adapting the open-chamber gun principle to a 37mm gun having a rate of fire of 10,800 rounds per minute within a weight of 450 pounds.

<u>Section 4</u> treats the interior ballistics of this system and shows that a muzzle velocity of 3000 feet per second can be obtained with a maximum pressure of 38,000 pounds per square inch.

<u>Section 5</u> covers the exterior ballistics of the 37mm mine type round and shows that proper gyroscopic stability is obtained at sea level conditions.

<u>Section 6</u> covers the kill probability computations under the specified combat conditions. It also gives results of computations based on varied dispersion and firing time parameters. It is shown that this system attains considerably higher kill probabilities at 2000 yards future range than the objective specified.

[redacted]

Section 7 is a brief treatment of the effect on the maintenance and supply problem of certain features of the open-chamber gun.

INTRODUCTION

INTRODUCTION

This study has as its objective the attainment of the following minimum kill probabilities at the combat conditions specified.

It is assumed that combat takes place at 20,000 feet altitude. The attacker's speed is equal to the fighter target speed of 1400 feet per second. The bomber target speed is 811 feet per second. The future range is assumed to be 2000 yards.

Three Combat Conditions are specified:

No. 1 -- Pursuit course attack against bomber -- $P_{K_1} = 0.59$

No. 2 -- Pursuit course attack against fighrer -- $P_{K_2} = 0.26$

No. 3 -- Collision course attack against bomber 45° off tail --

$P_{K_3} = 0.59$ salvo

$= 0.62$ ripple

The above kill probabilities are to be met with a system whose weight does not exceed 2000 pounds exclusive of fire control. The time of burst is limited to 1.0 second maximum.

The 37mm open-chamber gun and ammunition, proposed as a solution to the long range, high kill probability problem, has been selected by this contractor after considerable study of other weapons systems.

One bar to the development of an ultra-high firing rate weapon has been removed by actually firing 20mm ammunition in a gun containing a discontinuity in the chamber wall. This proves that it is possible to load ammunition transversely, thus eliminating reciprocating parts and inherent mechanical time delays.

The weapons system proposed in this report exceeds the objective kill probability requirements within the specified limitations of weight and firing time.

The kill probabilities attained for the three specified Combat Conditions at 2000 yards future range are as follows:

No. 1 -- (Fighter to bomber pursuit) -- $P_{K_1} = 0.72$

No. 2 -- (Fighter to fighter pursuit) -- $P_{K_2} = 0.56$

No. 3 -- (Fighter to bomber collision) -- $P_{K_3} = 0.74$

It has been concluded that this weapons system lends itself to simplified fire control since it is possible to obtain high kill probabilities on a pursuit course attack as well as a head-on approach. In the latter case opening fire may take place beyond 5000 yards range, minimizing the danger of collision.

1

HISTORICAL BACKGROUND

SECTION 1

HISTORICAL BACKGROUND

Definition

The open-chamber principle is based on a method by which it is possible to eliminate axial translation of ammunition into a gun chamber. It is a system whereby a round of ammunition is moved transversely rather than longitudinally into a firing chamber. This firing chamber is formed by two or more members which move transversely at the proper time to contain the round of ammunition prior to firing.

This principle may be applied to guns of superior ballistic performance. It is important that the essential differences be noted between the high pressure open chamber gun and those mechanisms employing transverse loading of ammunition using a round in which the cartridge case serves as a chamber. The T-131 boosted rocket, for example, firing in the T-110 launcher, is in the latter category and is not considered an open-chamber gun within the meaning of this study.

The advantages of a gun system based on this principle are due to the fact that movement of ammunition and gun parts is reduced to a minimum with consequent reduction of accelerations and forces. Automatic weapons are simplified by the elimination of reciprocating parts such as ramming and ejection devices, and as a result extremely high firing rates may be achieved.

Previous Designs and Experimental Work

Applications of the open-chamber principle have been tried in the past by various gun designers but the results have been uniformly disappointing. As far as is known, all previous approaches to this design principle have made use

of cartridge cases of circular cross section.

One of these previous experimental designs consisted of two sprocketed drums that formed a chamber at their junction around a circular cartridge case. A series of cams and locks was required to move around the periphery of each sprocket in order to form the chamber and in order to permit relative movement between the sprockets and to avoid moving sprocket centers. In order to test this basic design, the sprocket chamber was simulated by means of closely fitted cylindrical half-blocks that surrounded the cartridge case. These half-blocks were inserted into a heavy-walled tube with practically zero clearance. Upon firing, the cartridge case ruptured longitudinally at the seams formed by the half-blocks. The results of this test indicated that it was impractical to maintain a cartridge case intact when fired under high pressure in a discontinuous chamber, and it seemed apparent that a circumferentially continuous chamber wall was necessary to prevent longitudinal rupturing of the cartridge case. Therefore, it was concluded that a transversely loading high-performance gun was impractical, if not impossible, to achieve in practice.

It is interesting to note that the Union forces considered the use of a transversely loaded gun in the Civil War period. This was known as the Ager Coffee Mill Gun, invented by Wilson Ager. It was made in .58 caliber size and was designed for a firing rate of 120 rounds per minute. Generically, however, it could not be considered an open-chamber weapon, since each cartridge case consisted of a heavy-walled cylinder which, in effect, served as a firing chamber. In this respect it was similar to the T-131 rocket case. A full description (Reference 1) of this gun is given in Volume 1, Chapter 2, of "The Machine Gun" by George M. Chinn, Lt Col, USMC.

Non-Circular Cartridge Open-Chamber Gun

Since experimental evidence indicated that the open-chamber principle was impractical for use with a circular cartridge case, it was decided to investigate the possibility of use of some form of cartridge cross-section other than circular. The first design study, as far as we know, of a proposed gun of this type resulted in the arrangement shown in Figure 1-1 (Drawing No. 790030) dated 28 September 1951.

A non-circular cartridge case, which can be indexed, permits controlled reinforcement at the lines of chamber discontinuity. A thin-walled non-circular cylinder tends to assume a circular section under internal pressure before rupturing, the elasticity of the material determining the degree of approach toward a circular section. It was reasoned that if the non-circular cartridge case were made of a fairly elastic material, and if it were supported on all sides, a considerable amount of breech deflection could be tolerated without causing longitudinal tearing of the case. Furthermore, it was reasoned that since the case would be indexed within the chamber in certain fixed positions, it would be possible to reinforce the unsupported corners at the joints, and thus prevent extrusion of the corners into these gaps. This would be impractical in a circular case because there is no simple way of indexing it in fixed positions without adding exterior protrusions and bringing about other complications.

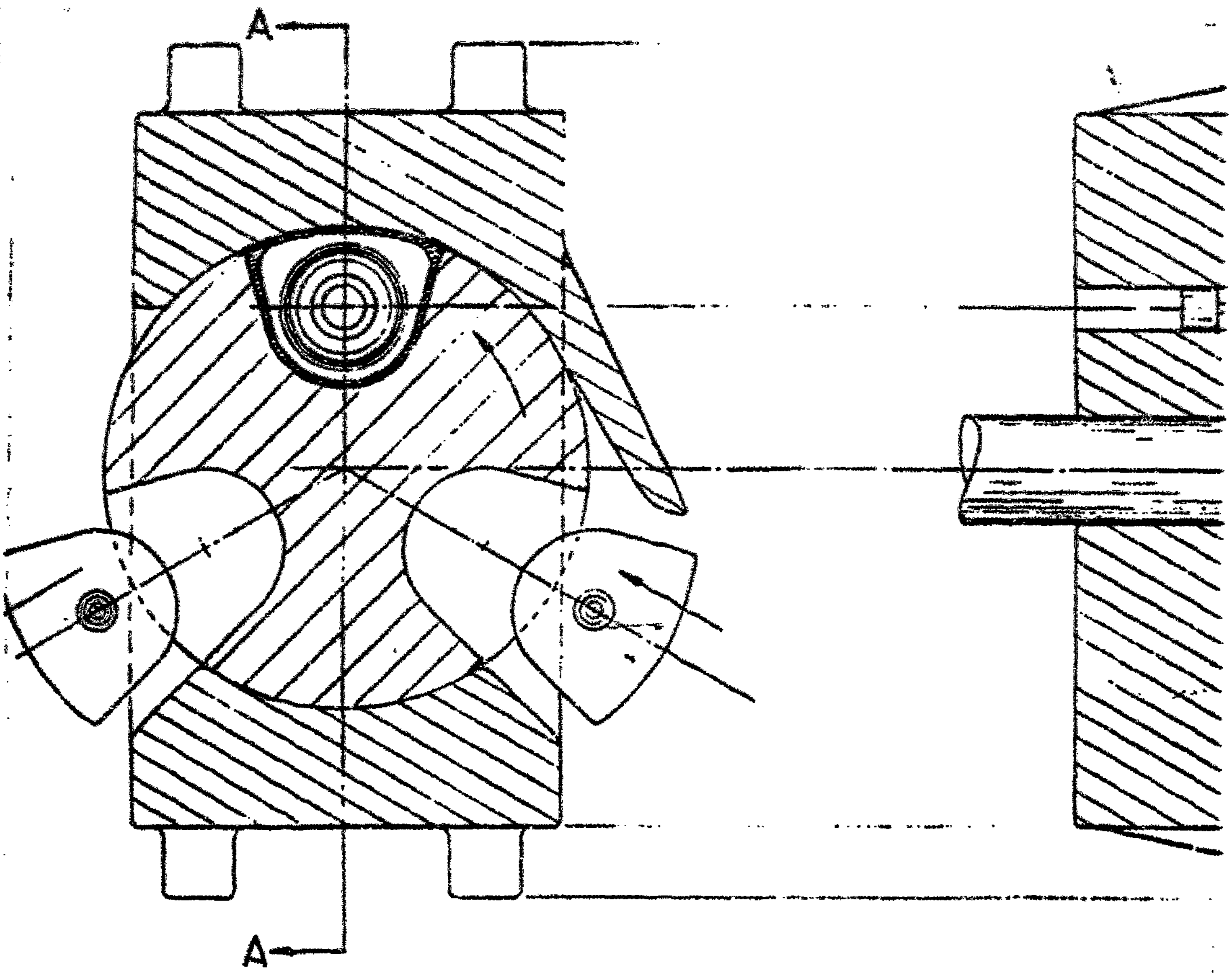
The basic gun design, as shown in Drawing No. 790030, uses a rotary drum which has one or more longitudinal recesses on its periphery. As each recess passes the stationary breech, the firing chamber is formed by the rotary drum and the breech. The round of ammunition, also shown on Drawing No. 790030, is shaped to conform to the chamber formed by the longitudinal recess in the drum

and the breech. The case shown on this sketch is approximately semi-circular in cross section, although it may be of any convenient outer shape such as a triangle, rectangle, etc. It is also seen that the corners are reinforced to prevent extrusion of case material into the gaps formed when gun parts deflect under high pressure.

An important element of the cartridge case design is the internal cylindrical sleeve used to hold the projectile. The sleeve and its front connecting flange form an obturating seal against the rear face of the barrel. Thus it may be seen that a completely obturated gun chambed is obtained by the cartridge case within the recess formed by the drum and breech.

The gun is loaded by inserting the ammunition from one side transversely into the drum recess. The drum then rotates until the ammunition is supported longitudinally by the stationary breech, at which point it is fired. The drum continues its rotation and the fired cartridge case is ejected transversely on the opposite side.

Using this basic principle, a large number of different gun configurations are possible. For example, the gun may be provided with one or more stationary barrels attached to the breech, the number of barrels being determined by the desired number of firing positions. In this case, the drum indexes into firing position opposite the barrel, or barrels, and remains stationary while the projectile is fired. Another possibility employs rotary barrels attached to the drum opposite each longitudinal recess. In this case, firing takes place as soon as the ammunition is covered by the stationary breech and it is unnecessary to hold the drum and barrels at rest during firing since they are in alignment at all times. The breech would be made wide enough to keep the case covered



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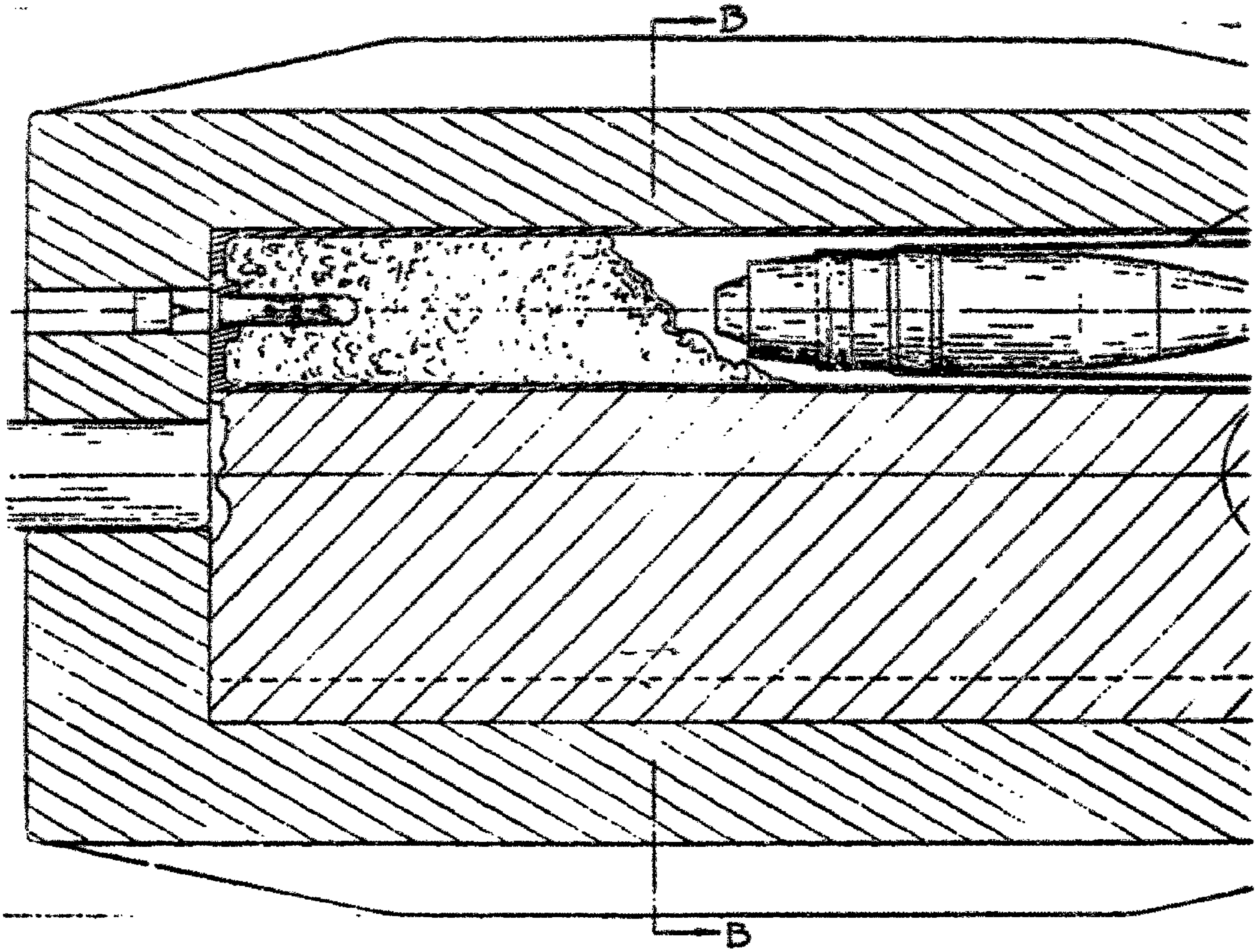

SECTION A-A

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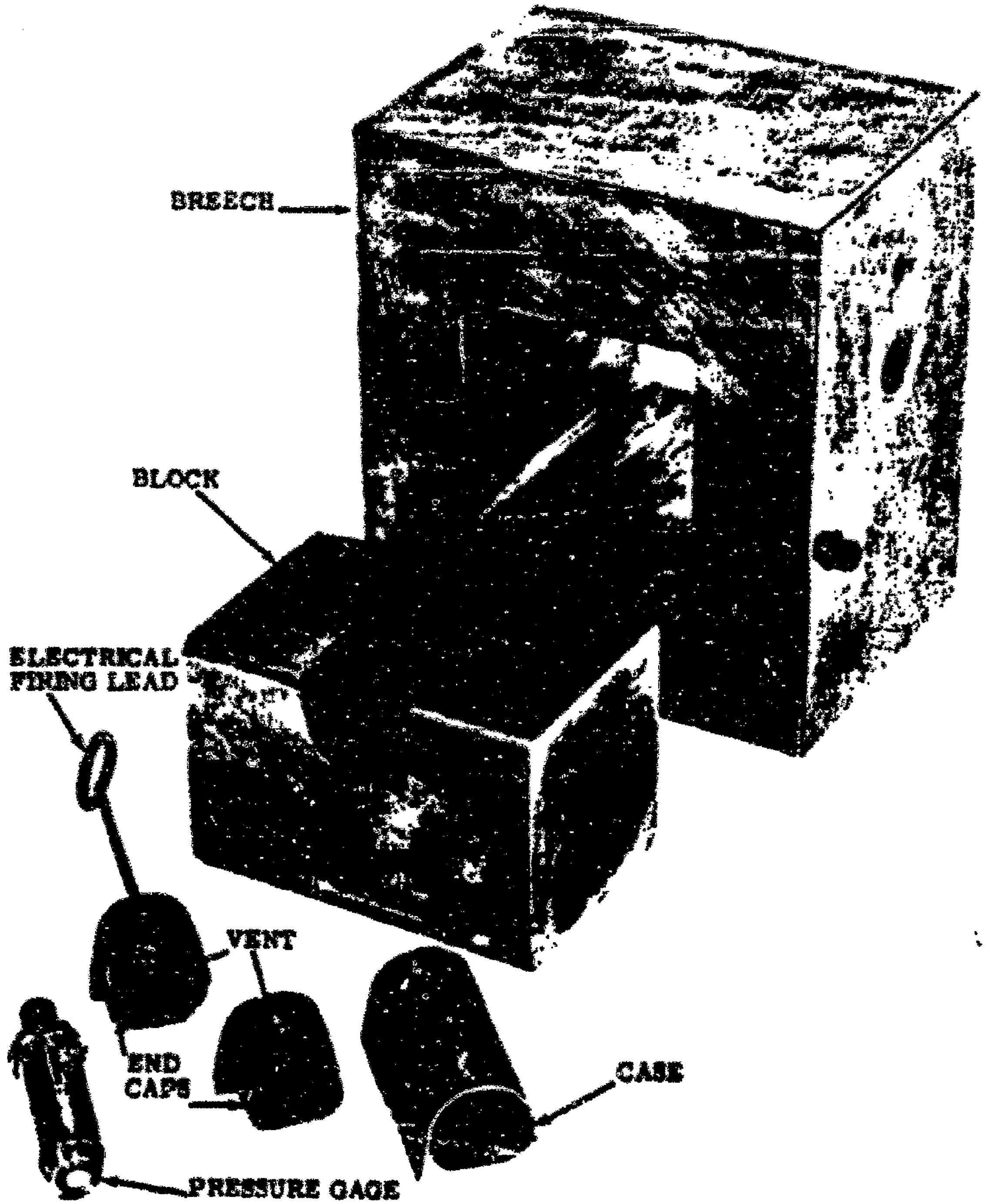


Figure 1-2. Disassembled Test Case and Chamber

Figure 1-3. Partly Disassembled Test Case in Test Block

for the several milliseconds during the period of high pressure. The case could be covered with an anti-friction compound, such as Teflon, to minimize the high friction loads that would otherwise be encountered. With this system it appears possible to attain extremely high firing rates with multiple barreled weapons. Present studies indicate the possibility of rates of fire in excess of 20,000 rounds per minute with guns of 30mm or even higher caliber.

Many actuating means may be used to rotate the drum or drum-and-barrel assembly. It is possible to use such power sources as gun gas, recoil, external electric, hydraulic or gas turbine motors, etc. The elimination of any major reciprocating parts brings about the possibility of using very simple drives with many variations of motive power.

A major advantage of the positively indexed round lies in the possibility of using pre-engraved rotating bands in order to eliminate engraving stresses and prolong barrel life. Proper indexing of pre-engraved rotating bands is difficult to achieve in a high rate of fire weapon with circular cases.

Test Chamber

In order to prove the basic feasibility of the open-chamber principle, a test chamber and two cartridge cases were fabricated in May 1952. One of the cases, with the disassembled test chamber, is shown in Figures 1-2 and 1-3. The two cases were machined from solid bar stock, one being made of SAE 1020 steel; the other of half-hard commercial brass. End caps equipped with small vent holes were provided for each case to permit the escape of powder gases. Provision was made for the insertion of a pressure gage at one end of the test breech to measure the internal pressure generated by the burning of the powder.

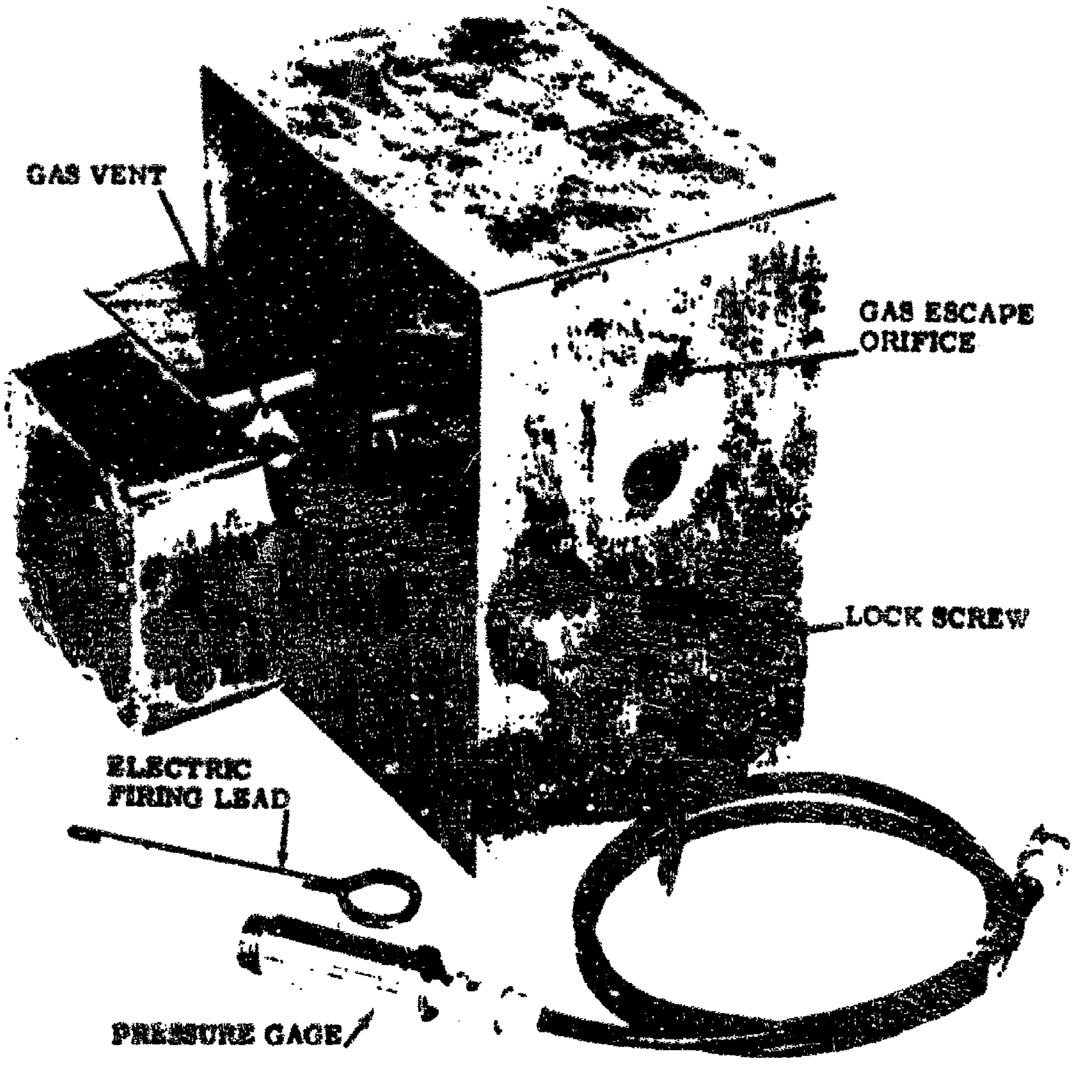


Figure 1-4. Assembled Test Case in Test Block

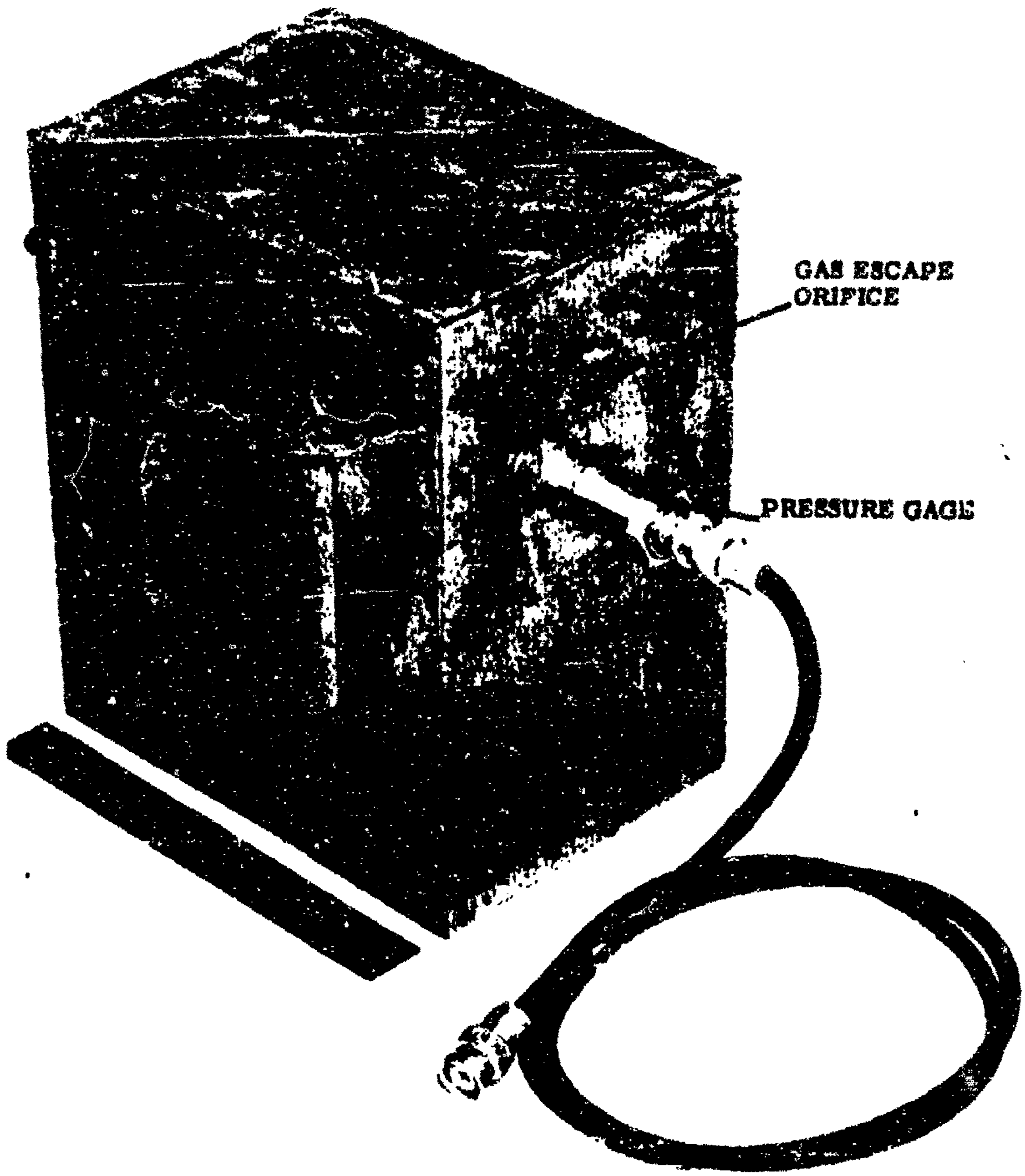


Figure 1-5. Assembled Test Chamber

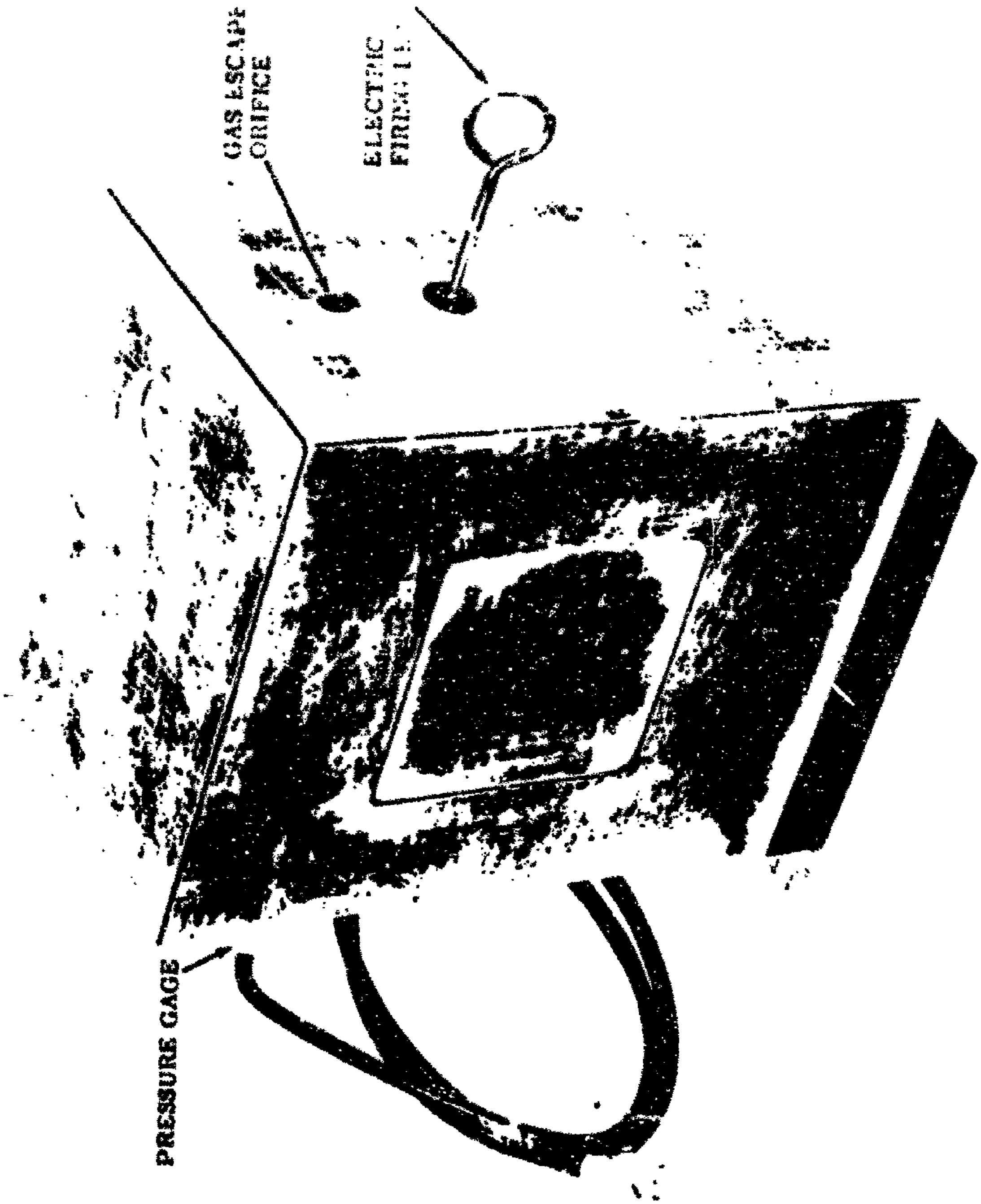


Figure 1-6. Assembled Test Chamber

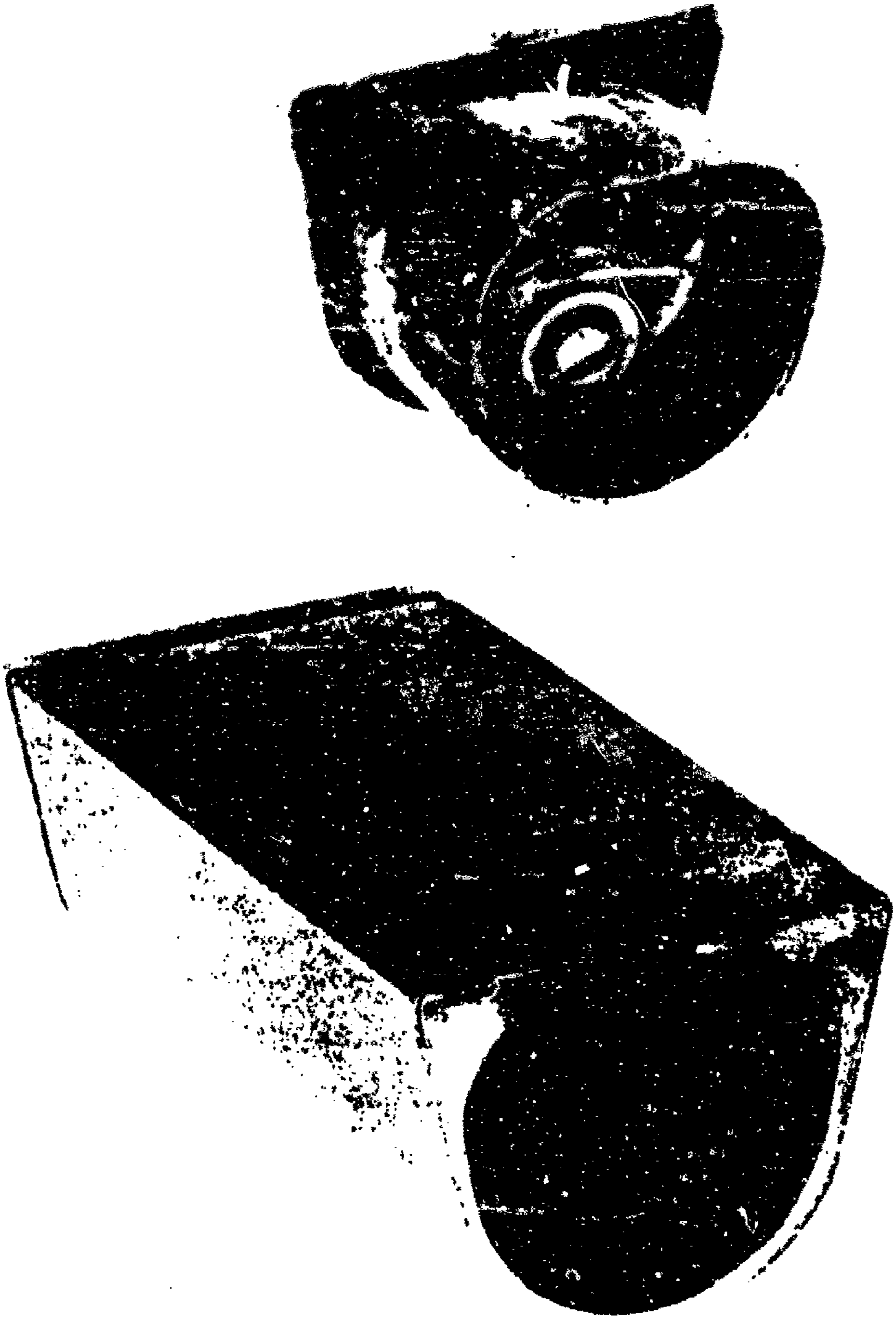


Figure 1-7. Brass Case After Firing

Figures 1-4 and 1-5 show the manner of assembling the test block, case, and pressure gage. Figure 1-6 shows the assembly of the electric firing lead that connects to an electric squib located within the case.

The initial firing tests, with the equipment described, took place on 2 September 1953. The cases were loaded with seven to eight grams of TMR 6962 powder removed from 20mm ammunition. The case volume was 26.9 cubic centimeters. The pressure recorded in both the steel and brass cases was approximately 45,000 psi. A subsequent examination of the cases revealed no failure of any sort. Figures 1-7 and 1-8 show the brass case after firing.

It is believed that these tests represent the first successful firing at high pressure of a cartridge case within a discontinuous chamber. On the basis of the results it was decided that continued development and tests, with a test gun, were warranted.

20mm Open-Chamber Test Gun

In order to determine how the interior ballistics of an open-chamber type gun would compare with the ballistics of conventional weapons, a test gun and special ammunition were made. No attempt was made to design an optimum gun in this instance, but merely to fabricate a firing chamber fitted with barrel and capable of firing a high-pressure round of ammunition without failure.

The gun is shown in Figure 1-9, Drawing no. 770610. It consists of a rectangular breech (similar to the one used in the original static firing test), a sliding block containing a triangular longitudinal recess, and a standard 20mm barrel. The barrel was modified by cutting off the portion containing the original chamber and threading the rear end to fit the rectangular breech. Since the upper block surface was curved to simulate the shape of an open-chamber

Figure 1-8. Brass Case After Firing

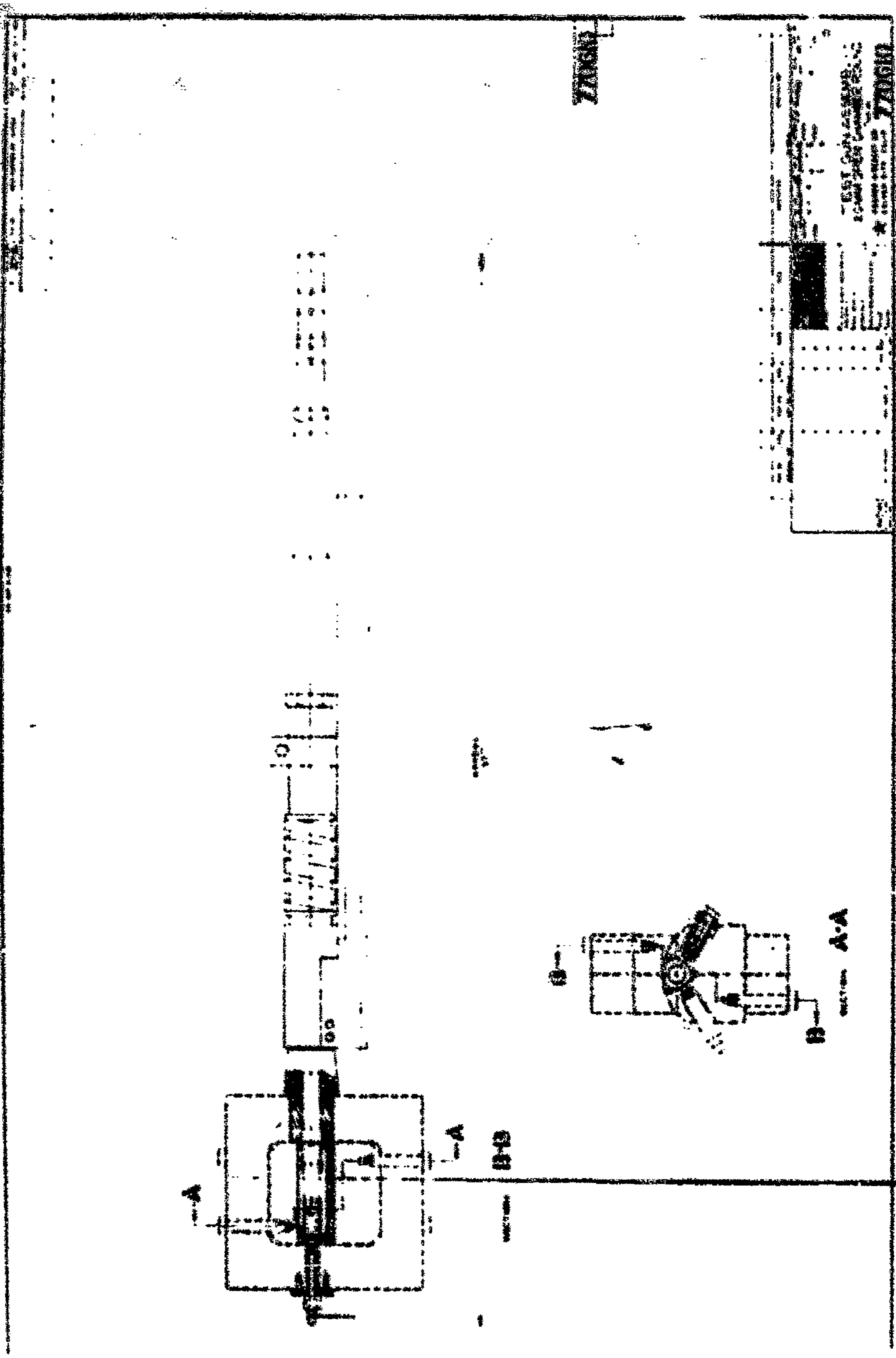


Figure 1-9. 20mm Open-Chamber Test Gun

drum, an auxiliary block was made to fit over it, covering the top of the round so that the entire assembly could be made to slide in and out of the rectangular breech recess. A conventional recoil adapter and a modified 20mm cradle were used to support the gun on a firing stand.

Figure 1-10 shows the entire gun assembly ready for firing. Two different types of pressure gages were used simultaneously to ascertain the maximum pressure. The one shown in Figure 1-10 is the same as the one used for the original open-chamber static firing test and is known as the Rutishauser type. Figure 1-11 shows this gage assembled in place. A copper crusher type gage was used on the opposite (right) side of the gun (shown in Figure 1-12). Copper cylinders and a Tarage table were obtained from Frankford Arsenal.

Figures 1-13 and 1-14 show the assembled and disassembled round of ammunition, respectively. The ammunition is of the telescoped type with the projectile entirely within the cartridge case and follows the basic design used for the special round designated as Design No. 4 (Reference 2).

The triangular-shaped rounds had the following characteristics:

Case material -- SAE 1020 steel

Sleeve material -- soft copper

Projectile -- M99, 2000 grains. Rear rotating band chamfered 15°

Volume of case -- 3.50 cubic inches (58 cubic centimeters)

Propellant -- 42.45 grams IMR 6962

Loading density -- 0.73

Primer -- Electric, 1/4 grain

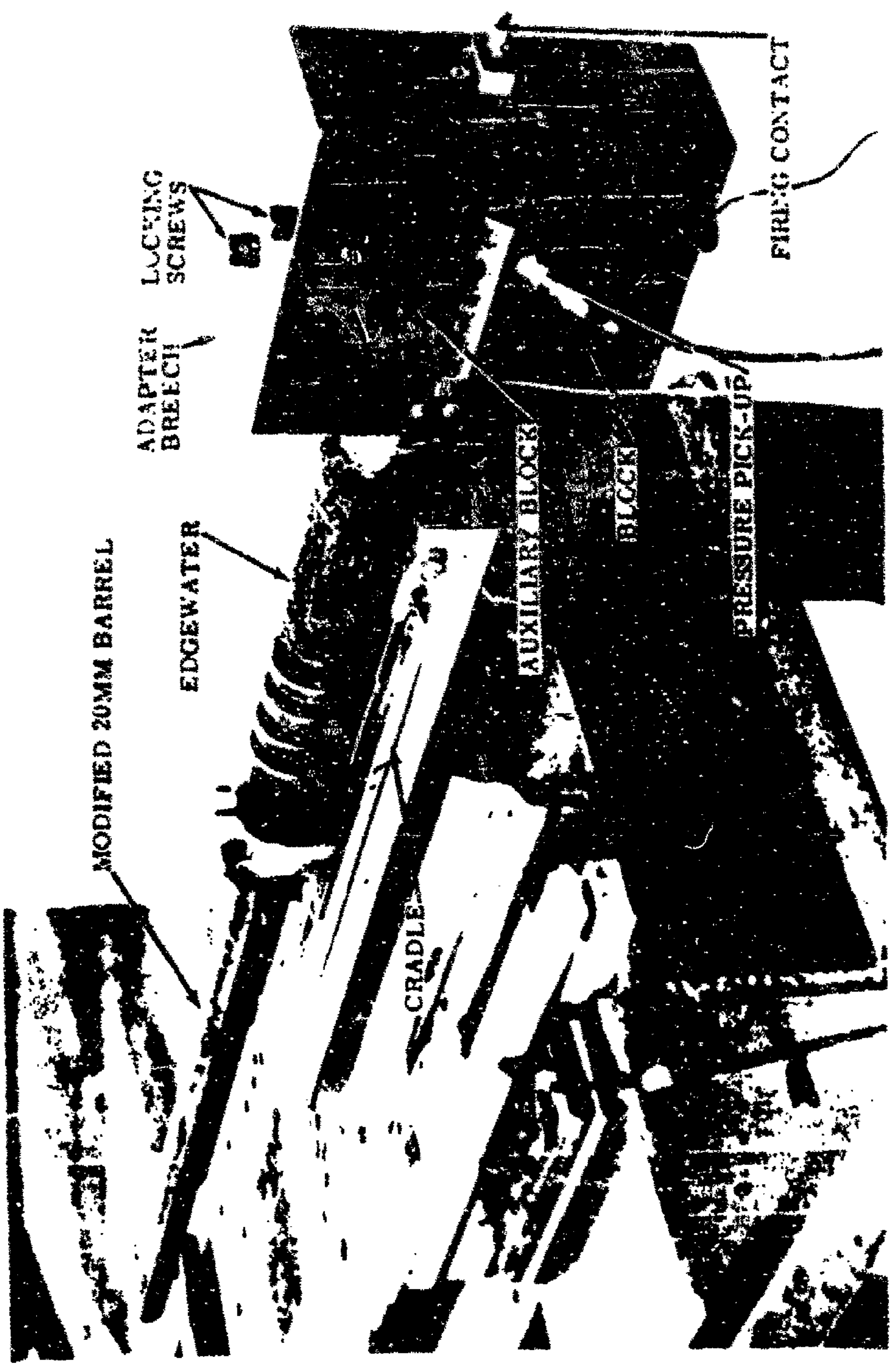


Figure 1-10. 20mm Open-Chamber Test Gun on Firing Stand

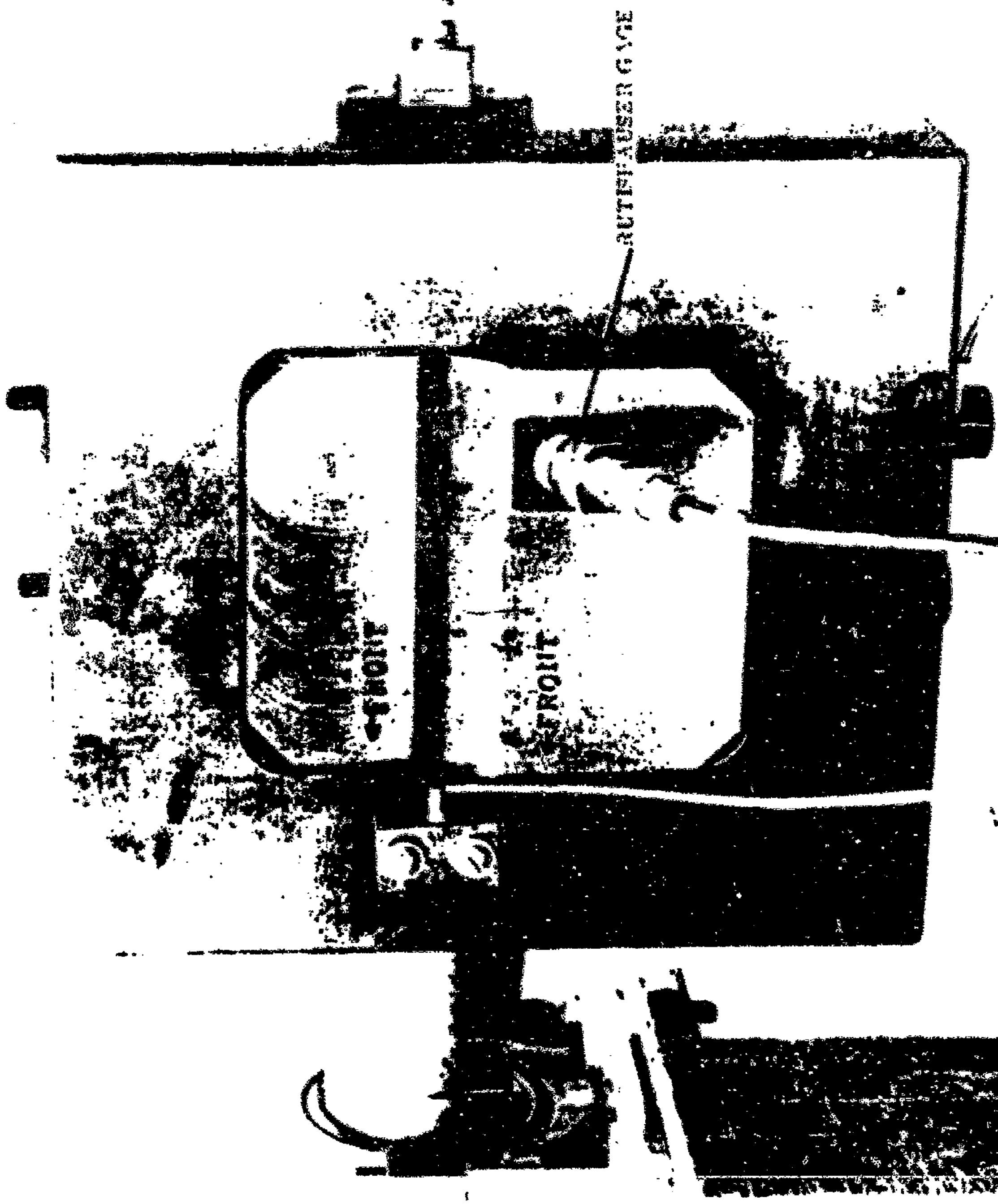


Figure 1-11. Closeup of Rutishauser Gage

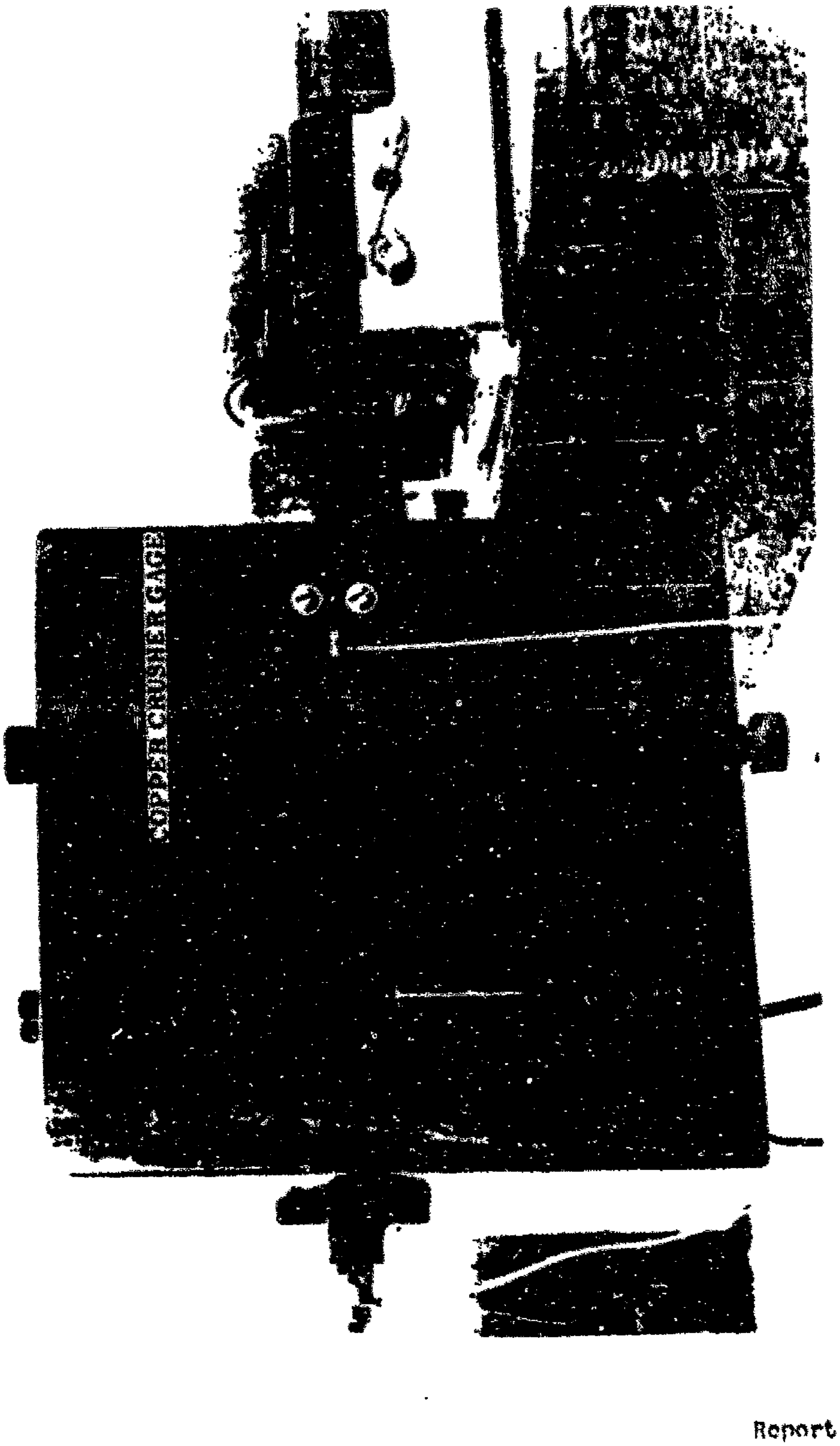


Figure 1-12. Closeup of Copper Crusher Gage

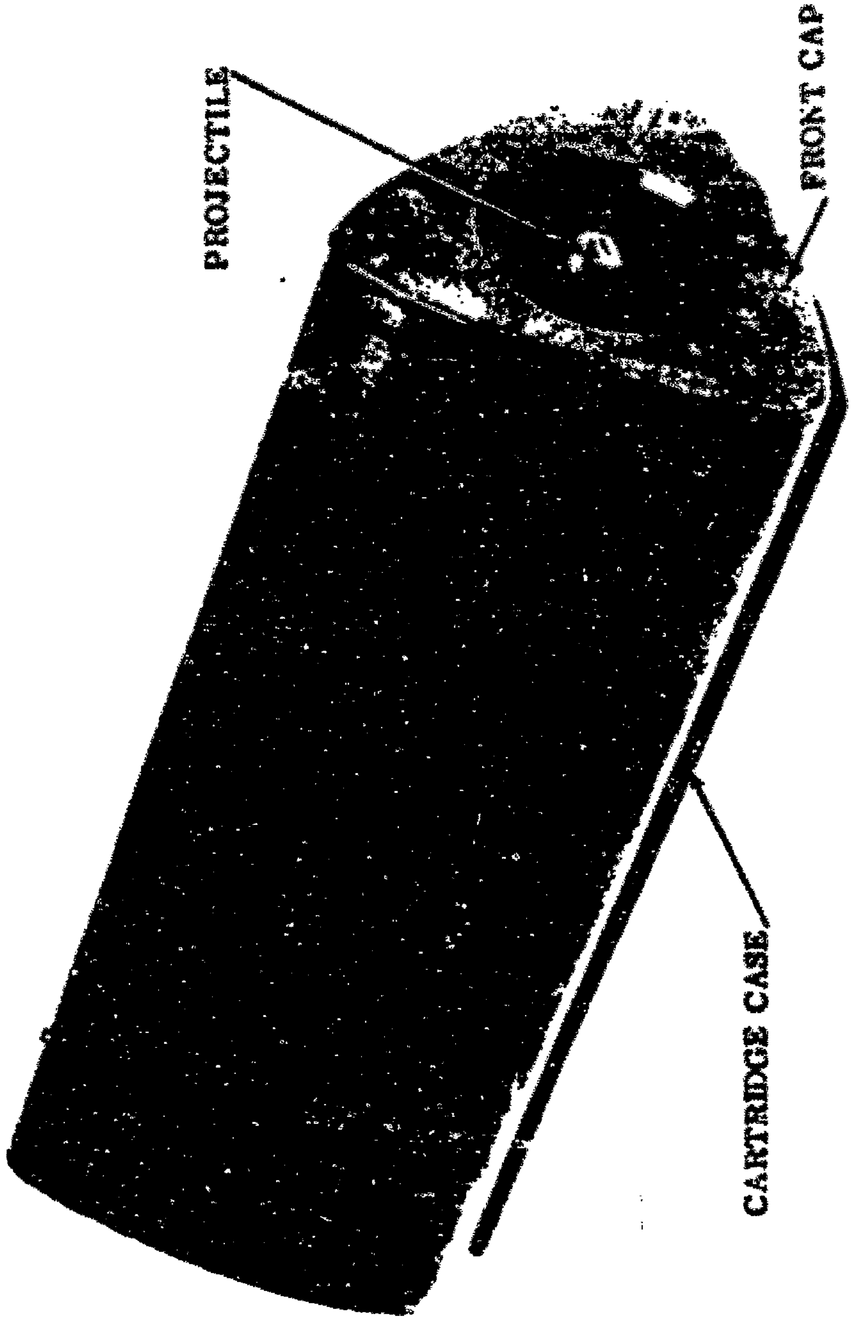


Figure 1-13. Assembled 70mm Triangular Round

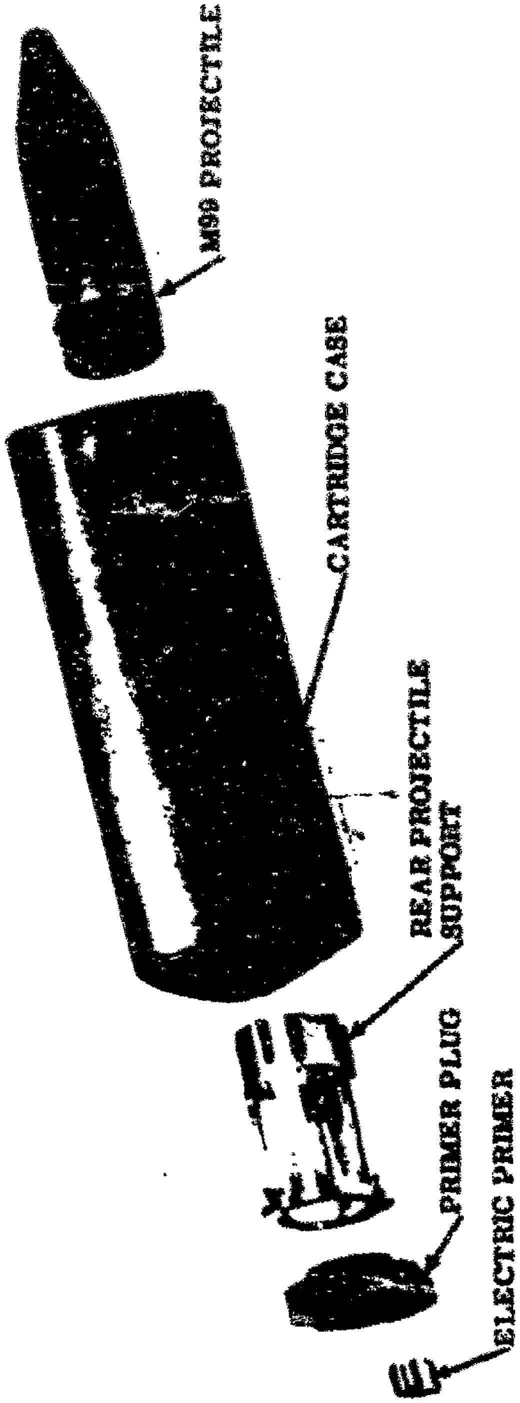


Figure 1-11. Disassembled 20mm Triangular Round

The projectile was held at the rear by the brass projectile support with a crimp into the projectile crimping groove to simulate the condition obtained with a conventional cartridge case. The front end cap, which carried the cylindrical front support for the projectile, was brazed to the cartridge case. The rear cap was cemented in place and contained a thin flange that provided obturation by expanding under pressure against the interior of the outer case.

The first firing took place on 25 November 1953. Only one shot was fired and the instrumental velocity of the projectile was 2620 feet per second, which corresponds closely to the predicted velocity of 2600 feet per second for the known loading conditions. A pressure-time curve, which indicated normal interior ballistics, was obtained with the Rutishauser gage. Study of the pressure gage readings indicated a maximum chamber pressure of approximately 40,000 psi. The cartridge case showed no evidence of rupture, either in a longitudinal or a transverse direction. The front cylindrical projectile support was damaged, apparently due to excessive rotating band interference.

After firing it was determined that the cartridge case had a permanent diametral deformation of 0.016 to 0.018 inch. Since the original clearance between receiver and breech was approximately 0.012 inch, this resulted in a tight assembly after firing. Considerable force was required to separate the sliding block assembly from the breech due to this interference condition. The gun parts showed no permanent set after firing and showed no indication of local over-stress conditions.

High-speed motion pictures taken during the firing test showed good obturation at the front and very little gas leakage at the rear. This slight

gas leakage could have occurred through the primer plug threads or around the flange of the rear end cap.

It is interesting to note that, as far as is known, this was the first shot ever fired without case failure under high-pressure conditions from a gun having a circumferential discontinuity in the chamber wall.

The test also showed the desirability of having a case material with a high degree of elasticity so that it can return to its original shape and so eliminate post-firing friction loads. This is not as important in an application where there is continuous movement of the case relative to the breech, such as the rotary barrel open-chamber designs. In this case, friction loads are minimized by use of Teflon or other anti-friction coats, and the dynamic friction is somewhat less than the static friction between the same materials.

The second firing test was conducted on 16 December 1953 using the cartridge case that was fired in the first test. This case was remachined to the original dimensions, giving it a thinner wall than previously. A new front cylindrical support was made, and the band interference condition was alleviated by an improved design of the copper sleeve. The same powder charge was used (42.45 grams) with a better powder distribution at the rear by use of a compressed paper filler around the nose bushing. The original case-breech clearance of 0.012 inch was reduced to approximately 0.002 inch. No velocity measurement was obtained due to failure of the recording apparatus. The indicated maximum pressure was again approximately 40,000 psi.

The lateral static force required to separate the block assembly in this firing was about one-half of that required in the first test. Diametral permanent expansion of the cartridge case occurred. The front end cap, cylindrical

support, and case body were undamaged. The rear end cap was split longitudinally, possibly because of excessive expansion in the first firing test and subsequent failure since the material was in a pre-stressed condition.

High-speed motion pictures taken during the second firing test indicated more gas leakage than in the first test due to a somewhat different assembly condition. In this case, the front end cap was not brazed to the cartridge case but was simply a light push fit within it. Hence the case expanded around the end cap permitting a gap to form and gas to escape. As in the case of the first test, the gun parts showed no permanent deformation.

A third firing test was conducted on 13 January 1954 using an aluminum cartridge case of 75S-T composition. This material has a high elastic limit but very little elongation, and it was anticipated that the case might fail under excessive deflection. Upon firing, the case failed longitudinally in two places, permitting a gas leakage and consequent erosion of the breech parts. This series of tests indicate that it is more important that the case material possess a high elongation rather than a high yield point.

For this third test the charge consisted of 42.8 grams of IMR 6962 powder with a case volume of 59 cubic centimeters. The recorded instrumental velocity was 2330 feet per second. The maximum pressure recorded was approximately 34,000 psi. Very little force was required to separate the block assembly from the breech after firing.

It is apparent that an intensive program of materials research and development would be highly desirable for application to cartridge cases for open-chamber guns. This should include investigation of non-metallic materials such

as fiberglas and laminated phenolics, as well as combustible materials such as nitro-cellulose. In the case of the latter, various obturating means must be investigated.

References

1. George M. Chinn, Lt Col, USMC, "The Machine Gun," Bureau of Ordnance, Department of the Navy, 1951.

2. Hughes Aircraft Company Report No. 09-180-D, "30mm Aircraft Gun T154," Contract DA-04-495-Ord 415, pp 9-15, Drawing No. 770408.

2

INVESTIGATION OF EXISTING WEAPONS PRINCIPLES

SECTION 2

INVESTIGATION OF EXISTING WEAPONS PRINCIPLES

Combat Conditions

In order to meet the basic requirements of the 30mm to 70mm Weapons Study, it is necessary to consider only systems that equal or exceed the specified minimum kill probabilities under the specified conditions. Further, the weight of the weapon and ammunition must not exceed 2000 pounds.

As stated previously, the minimum kill probabilities are 0.59 and 0.26 against a bomber and fighter respectively on a pursuit course attack, and 0.62 against a bomber on a collision course 45° off the tail. These kill probabilities must be achieved at a future range of 2000 yards, which is defined as the distance between the attacker and target at the moment of projectile impact. Present range is defined as the distance between the attacker and target when the projectile is launched or fired. It is obvious that, in the case of the fighter target, and targets of equal speeds, present and future ranges are identical.

The combat conditions are assumed at 20,000 feet altitude. The bomber target speed is 811 feet per second while the speeds of the attacker and the fighter target are assumed to be equal at 1400 feet per second.

Parameters Required to Meet Kill Probabilities

The minimum kill probabilities have been computed by the Ballistic Research Laboratory for the following conditions:

Number of projectiles fired	100
Explosive content per projectile	1.0 lb TNT
Weight of projectile	5.0 lb

Time of flight to target at 2000 yards
future range .... 2.0 seconds

Time to fire on pursuit course attack .... 1.0 second

Time to fire on collision course attack .... 0.3 second

Any of the above parameters could vary provided that the relative striking velocity of the projectile exceeds 1400 feet per second to ensure proper fuze functioning.

The firing time must be held to a maximum of 1.0 second in order to meet conditions brought about by targets of opportunity and to meet maximum survival probability against a bomber carrying effective defensive armament.

The weapons system specifications, including ammunition but not fire control equipment, are as follows:

TABLE 2-1

Item	Maximum Limits
Volume	40 cubic feet
Weight	2000 pounds
Length	84 inches
Recoil	40,000 pounds

In addition, no external stores are permitted, except for retractable launcher pods similar to the one on the F86D.

Method of Approach

In considering the possible variations of the outlined projectile and weapon parameters, it was realized that extremely high firing rates would be required, in addition to relatively short time of flight.

The time of flight of 2.0 seconds means air travel of 8800 feet after launching or firing, before impact, at 2000 yards future range. The average velocity of 4400 feet per second would be difficult to obtain by means of conventional propellants. Therefore, it was clear that some of the other parameters would require modification in order to relax the time of flight requirements.

Since firing time was fixed at a 1.0 second maximum for a pursuit course attack, there were only two significant parameters left for consideration: explosive content, and number of projectiles fired.

A study of the vulnerable area curves in BRL Note 807 shows that a fighter target requires a maximum of 0.6 pound of TNT on a pursuit attack. Beyond that, no purpose would be served by increasing the H.E. content since a condition of "overkill" would be obtained.

The vulnerability curves for the bomber target show that the vulnerable area is not linear with H.E. content of the projectile in the range considered.

Therefore, it was concluded that it would be most advantageous to increase the number of projectiles fired, and decrease the H.E. content, in order to compensate for an increased time of flight.

The following modified system characteristics were set up as a first approximation toward a solution:

Number of projectiles fired .... 200 minimum

H.E. content (minimum TNT equivalent) .... 0.50 pound

Time of flight to 2000 yards
future range .... 2.5 to 3.0 seconds

Cons[illegible]ati[illegible] of Weapon Types

Three [illegible] types of missiles may be considered as possible solutions to the problem of meeting the specified kill probabilities:

R[illegible]ets propelled by rocket motors only;

Boosted rockets which are launched or fired out of a gun barrel with a substantial initial velocity that is increased to a higher terminal velocity by means of a rocket motor;

Gun-fired projectiles having no additional rocket power.

Rockets. A study of existing rockets showed that the desirable size would be in the 1.5 to 2.0 inch diameter class. The weight of the rocket was assumed to be between 3 and 6 pounds. A brief analysis narrowed the optimum size down to approximately 1.5 to 1.75 inches.

It was concluded that while a rocket in this size range would meet the kill probability requirements up to 1000 yards, it would be virtually impossible to meet the conditions at 2000 yards future range. To do so it would require a burnt velocity far higher than may be achieved with propellants known to this Contractor.

Boosted Rockets. The rockets considered in this class were also assumed to be between 1.5 and 2.0 inch in size. Either fin or spin stability could be considered since it was assumed that a muzzle velocity of approximately 1000 feet per second would be obtained.

Spin stability was eliminated as a possibility in the 1.5 inch size because it would be very difficult, if not impossible, to have a warhead of the required H.E. content and still maintain a length/diameter ratio small enough for proper stability.

However, a spin-stabilized, 2-inch, boosted rocket was considered as a possibility. It was reasoned that since the 2.75-inch T-131 rocket weighed 5.2 pounds with 1.0 pound of explosive, it would be possible to obtain a similar 2-inch rocket for about 3.5 pounds with 0.5 pound of H.E. filler. However, the burst weight would not be significantly higher than that of the 1.5-inch rocket, and the exterior ballistics would therefore be inferior, since the ballistic coefficient would be lower by a factor of d_1^2/d_2^2 where d_1 is the diameter of the 1.5-inch and d_2 of the 2.0-inch rocket. This assumes that the drag coefficient would be equal in both cases. The fin-stabilized rocket would no doubt have a somewhat higher drag coefficient because of fin resistance.

Another reason for considering the 1.5-inch rocket is that less frontal area would be required for launching tubes.

It was decided that individual launching tubes would be preferable to automatic launchers, since it would be quite difficult to achieve the required firing rate with the latter. Moreover, a fin-stabilized rocket is fairly long and does not require much more than double its own length of travel in order to achieve a muzzle velocity of approximately 1000 feet per second with an average pressure of 6000 pounds per square inch.

For simplicity, it may be assumed that a 4.65-foot tube is used to obtain 1000 feet per second muzzle velocity. With an average pressure of 6000 pounds per square inch,

$$F = 6000 \times 1.77 = 10,700 \text{ pounds.}$$

Assuming the rocket weighs 3.25 pounds or has a mass of 0.10 slugs,

$$a = \frac{F}{m} = \frac{10,700}{.10} = 107,000 \text{ ft/sec}^2$$

For a travel of 4.65 feet, the time, t, in the bore is

$$t = \sqrt{\frac{9.30}{107,000}} = .00935 \text{ second}$$

$$V = 107,000 \times .00935 = 1000 \text{ feet per second}$$

That is, the average pressure of 6000 psi would suffice to give approximately 1000 feet per second muzzle velocity.

The maximum pressure would be on the order of 10,000 psi.

Assuming an average pressure of 6000 psi, the average wall thickness for 70,000 psi yield point would be

$$\frac{6000 \times 1.5}{70,000 \times 2} = .064 \text{ inch}$$

An aluminum tube 60 inches long, with the average wall thickness stated above, would weigh approximately 2.0 pounds.

Assuming a booster propellant weight of 0.5 pound and 1.5 for the rest of the tube assembly (end cap, primer, and other accessories) there would be a total weight of 4.0 pounds per round in launcher equipment exclusive of retractable pods.

Assuming the rocket to weigh 3.5 pounds, the total weight would be

4.0 + 3.5, or 7.5 pounds per round.

The ammunition and launcher load of 200 rounds would weigh approximately 1500 pounds, which would leave a comfortable margin of weight for pod mechanism and structure.

A brief study indicates that the time of flight for the boosted rocket would be approximately 2.4 seconds to 2000 yards future range. This is based upon a drag coefficient of 0.50 and a burnt terminal velocity of 5000 feet per second, which is arrived at as follows:

Muzzle velocity	1000 feet per second
Plane speed	1400 feet per second
Rocket assist	2600 feet per second
Total	5000 feet per second

Brief analysis indicates that this configuration would be capable of meeting the kill probability requirements. However, it is possible that the drag coefficient of 0.50 is rather optimistic for a rocket of this size; this would influence kill probability by increasing the time of flight.

Gun-Fired Projectiles. The previous rocket analysis indicates that a projectile of approximately 1.5 inches diameter carrying 0.40 pound of H.E. would tend to approach optimum conditions for meeting kill probabilities with minimum total weight of installation. This pointed to a 37mm size projectile.

In considering methods of firing at least 200 37mm projectiles in 1.0 second, several approaches were considered besides the open-chamber principle. These included the "squirt-gun" or Zettl principle of firing several projectiles at one time out of a single charge container. The principle of firing longitudinally stacked projectiles out of multiple barrel installations was also considered.

The conclusion was reached that the open-chamber principle was more applicable to this study than other principles as far as guns are concerned.

3

DESIGN CONSIDERATIONS OF GUN AND AMMUNITION

SECTION 3

DESIGN CONSIDERATIONS OF GUN AND AMMUNITION

Characteristics

Gun

Caliber 37mm

Overall Length .... 72 inches

Overall Height .... 16 inches

Overall Width (Without Drive) .... 12.5 inches

Number of Barrels 4

Gun Type .... Open Chamber

Operation .... External Drive

Rate of Fire .... 10,800 Rounds per Minute

Total Weight (Including Drive) .... 425 Pounds (Approx)

Total Weight (Without Drive) .... 350 Pounds (Approx)

Muzzle Velocity .... 3,000 Feet per Second

Maximum Powder Pressure .... 38,000 psi (Approx)

Rifling .... Uniform Twist -- 1 Turn in 25 Calibers

Recoil Distance .... 0.86 Inch

Recoil Load .... Under 15,000 Pounds

Ammunition

Weight of Projectile .... 1.35 Pounds

Weight of Powder Charge .... 0.46 Pound

Weight of Complete Round (Including Link) .... 2.50 Pounds

Type of Powder .... Double Base Composition

Type of Projectile .... H.E. (Mine)

Weight of H.E. (MOX or HEX) .... 0.40 Pound

Type of Rotating Band .... Pre-engraved

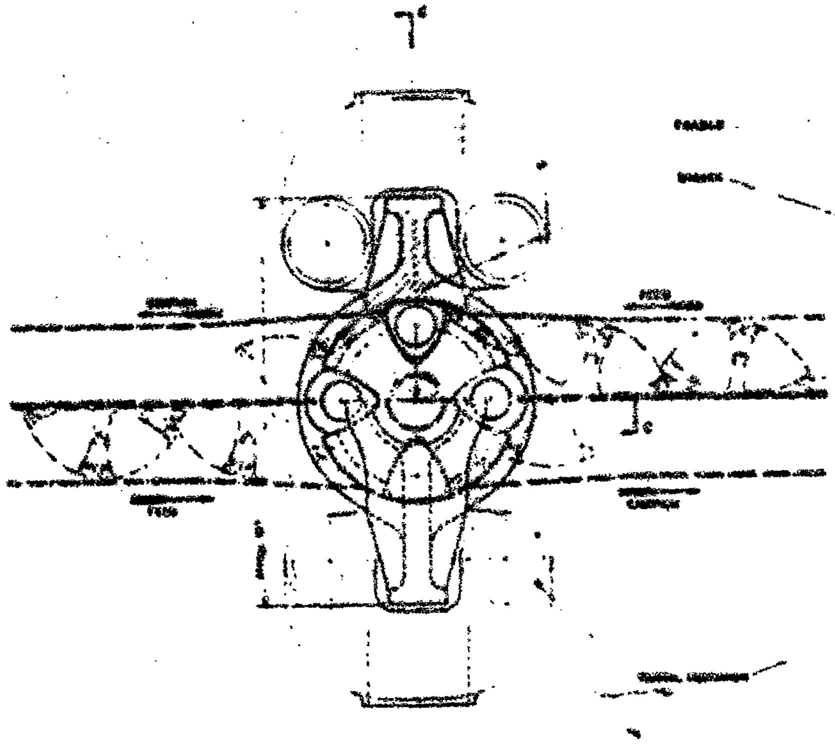
General Description

The general configurations of the 37mm Open Chamber Gun and ammunition are shown in Drawing No. 790319 and in Figure 3-1. This gun is equipped with four barrels attached to a rotating drum by interrupted threads or some other quick-locking means. The rotating drum has four longitudinal recesses — one opposite each barrel. The stationary breech, which provides a front and rear bearing for the drum assembly, forms a heavy beam structure at the top and bottom. The beam structures (shown in Section A-A of Drawing No. 790319), with the longitudinal drum recesses, form the firing chamber.

Each time the drum and barrel assembly rotate opposite the stationary breech, the ammunition is fired electrically. With the combination of four barrels and two firing positions, eight shots are fired at each revolution of the drum.

A back plate, which is rigidly fastened to the drum assembly, supports the cartridge case axially during firing. This plate carries a firing pin opposite each recess that makes a wiping contact as the firing position is reached. A fixed cam, attached to the breech, forces the firing pin forward against the electric primer at the proper time. Since the firing system is simple and of conventional design, it is not shown on this drawing.

The ammunition is linked together by collapsible hinge-like flat strips. There are two firing positions; therefore two belts are fed transversely, one



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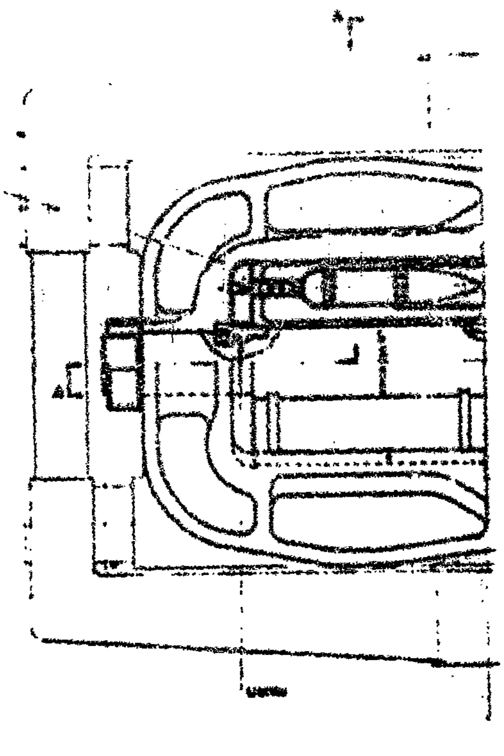

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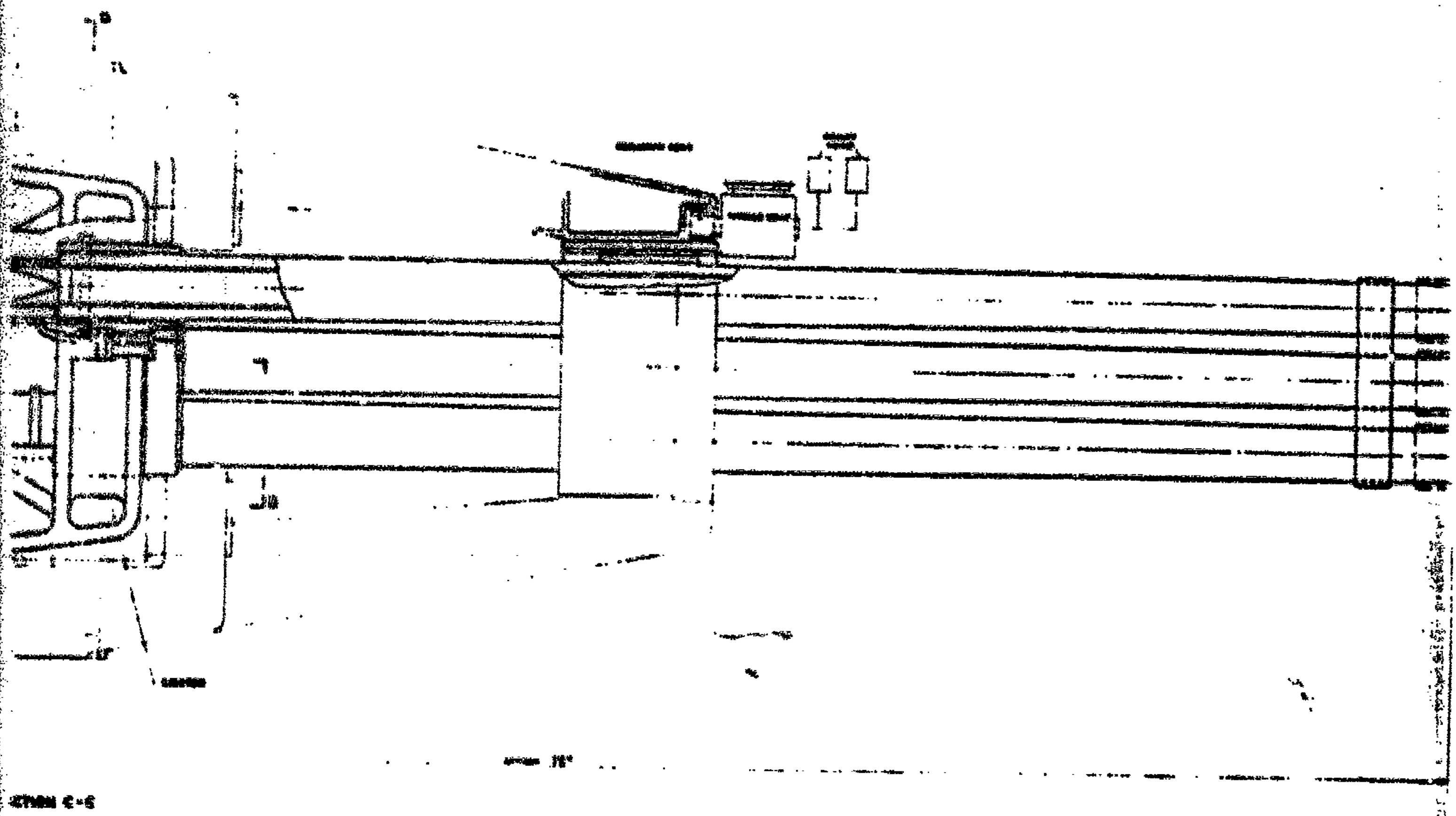

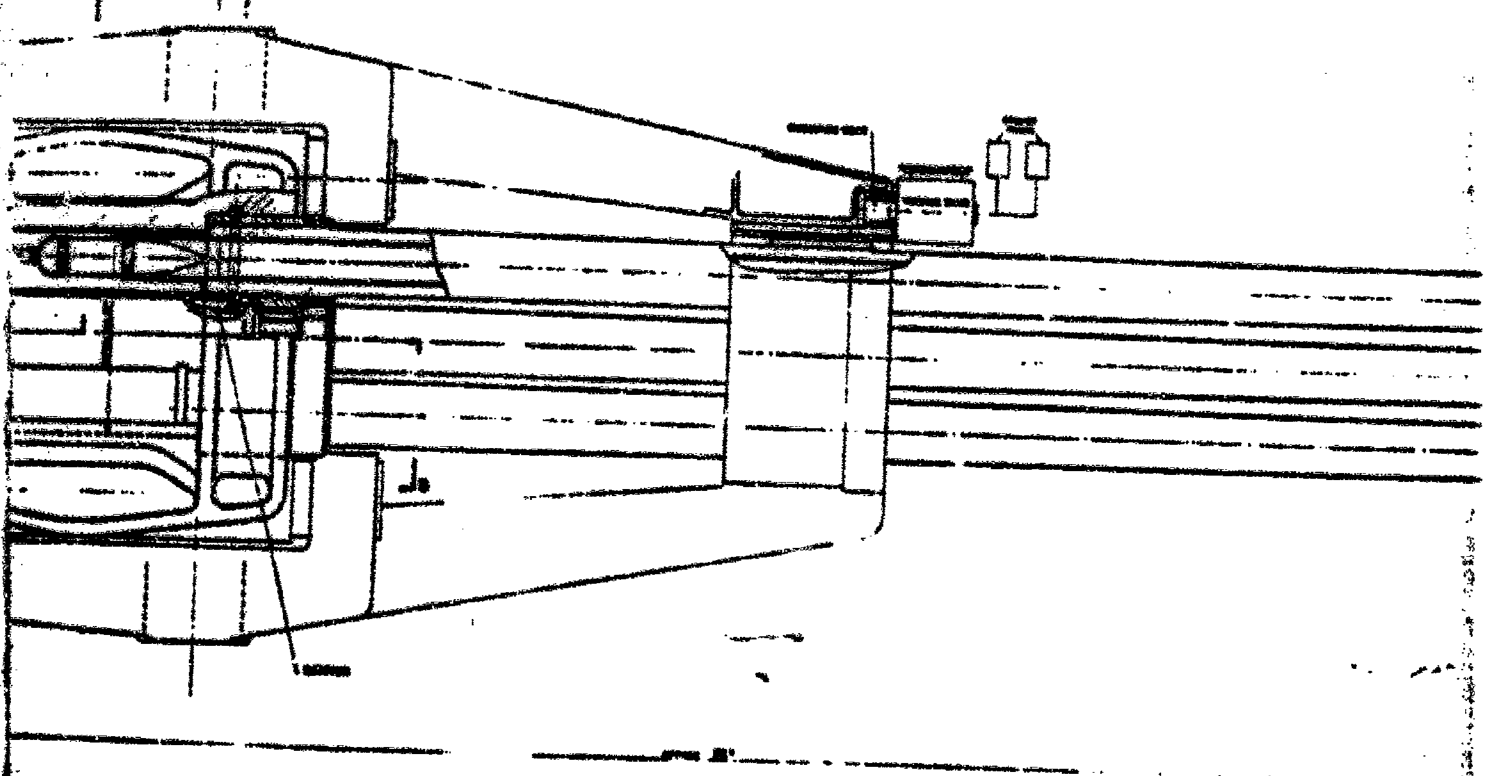

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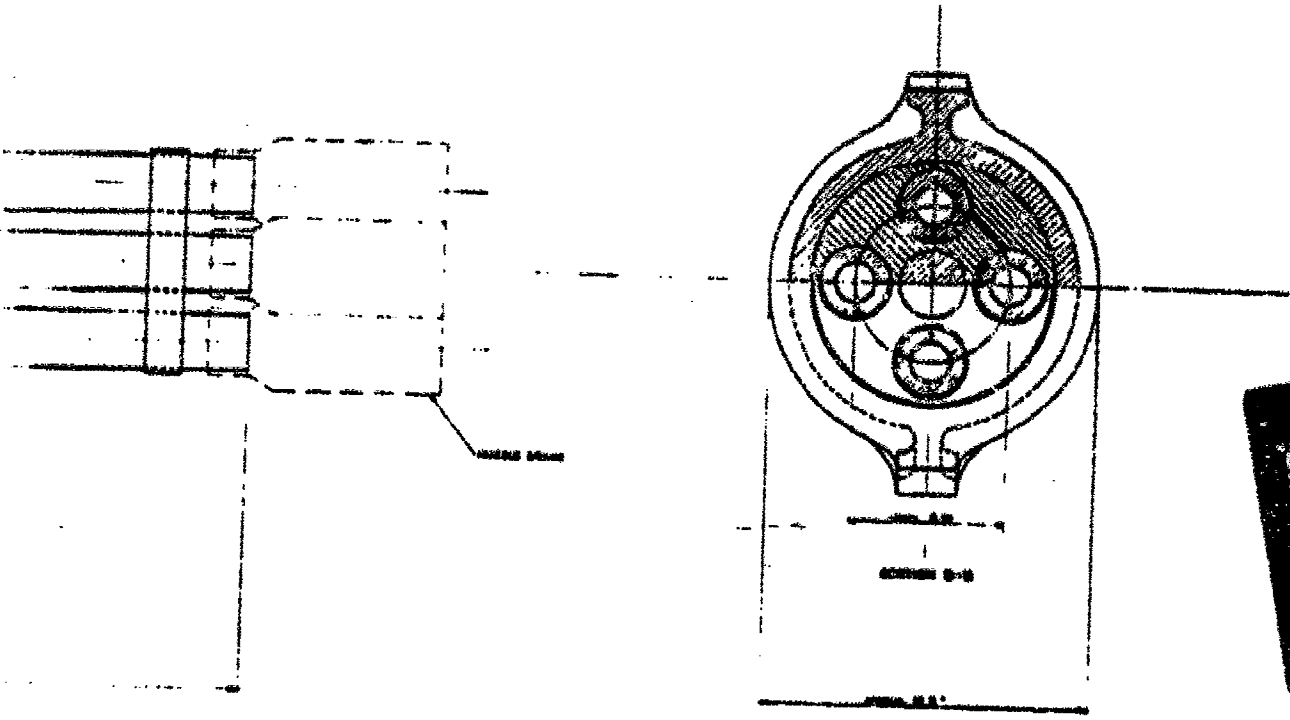

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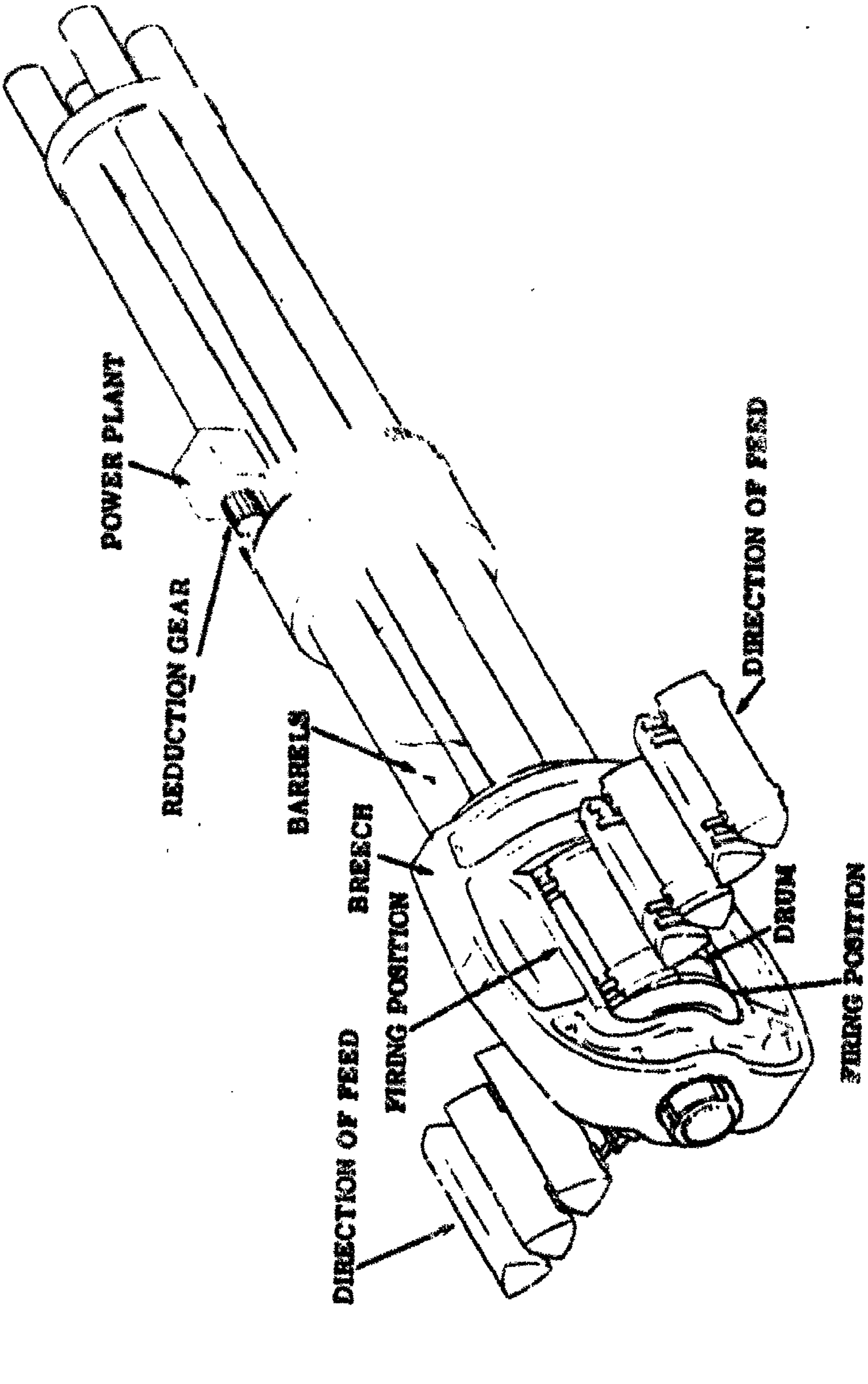


Figure 3-1. Schematic View of 37mm Open Chamber Aircraft Gun

at the top and one at the bottom. As soon as the fired case is uncovered during continuous rotation, it is ejected radially by centrifugal force and by two ejectors, which are located at the bottom of each of the longitudinal drum grooves at the front and rear of the case. The ejectors are actuated by fixed cams located in the breech.

As shown, the cartridge case is triangular in section. This shape requires minimum storage volume since two rounds may be stacked so that the bases of the triangles are on opposite sides, thereby forming a trapezoidal section. Hence, there would be practically no waste space in a storage box.

The following equation expresses the theoretical maximum number of rounds that could be stored in a box:

$$N = 2 \frac{A}{bh}$$

where

N = number of rounds

A = cross-sectional area of box

b = width of base of triangular case

h = height of triangular section

This number would be reduced somewhat by the space required for the linking system and by the fact that the rounds have a modified triangular cross section.

The drum and barrel assembly are rotated by an external power source. Although the horsepower requirements for driving the assembly are fairly high, they can easily be met by a compact, light-weight, short-life gas turbine operating at high speed. The gases for turbine operation could be furnished by cartridges, rocket motors, or some similar means. Gun gas might also be used to sustain an initial rotation started by a small power unit. A reduction gear would be used to obtain the required rotational velocity of the gun.

Before firing, the gun is brought up to speed within a short period of time (for example, 0.10 to 0.20 second). During this acceleration period, the ammunition belts remain stationary. When firing takes place, the firing circuit is energized; simultaneously a solenoid-operated pawl forces the first round against the rotating drum. As soon as a drum recess appears opposite the first round, the round starts to enter and is carried around by the drum to the firing position. The round does not have to enter the recess completely since a fixed feeding cam, attached to the breech, guides the round into the recess as it is carried around by the drum.

Since the belt links are collapsed, only one round at a time is accelerated which precludes high belt loads. Grooves are provided in the drum so that ammunition links may pass through the gun without delinking. The collapsed links are analogous to the couplings of individual cars on a long freight train that are positioned so that a locomotive may accelerate one car at a time. It is possible to conceive of a number of possibilities for cartridge case disposal. They may be delinked after firing by means of pulling out the hinge pins if it is desired to dispose of them overboard, or they may be conveyed back to the original storage space.

Because the ammunition belts remain at rest while the gun is being accelerated, it is possible to fire one belt and keep the other in reserve. However, once a belt is engaged, it must be fired out since it takes only 1.0 second to fire the 90 rounds.

Wooden Mockup

In order to determine the feasibility of using collapsible ammunition links or hinges, and of feeding the first round when the gun has reached proper rotational velocity, a full-scale wooden mockup was made of the gun and ammunition belt (see Figures 3-2 through 3-6). The drum and barrel assembly is rotated by means of the handcrank. The first-round actuator simulates the solenoid-operated mechanism of the actual gun. The actuator strips, which pass through the drum link slots, are fastened to a pulling handle at the opposite side of the gun.

While the revolving parts are being accelerated by the handcrank, the collapsed ammunition belt remains stationary. When these parts attain the desired rate, the feed operator snaps the first-round actuator handle, thereby engaging the first round and feeding the entire belt. The spring-steel fixed feed cams, shown in Figure 3-5, guide the wooden ammunition into the drum recesses. This wooden mockup gun has been successfully operated at 250 rpm which corresponds to a firing rate of 2000 rounds per minute.

Slow-motion films taken of the belt action demonstrate the action of the collapsed links as well as the method of engagement of the first round. Figure 3-6 shows the belt in the collapsed and the extended link positions. It is believed that with the use of Teflon-covered cartridge cases, the low coefficient of friction of the actual ammunition will permit successful first-round engagement at the rotational speeds required to meet the design objectives.

Figure 3-2. 37mm Mockup, Right Side View

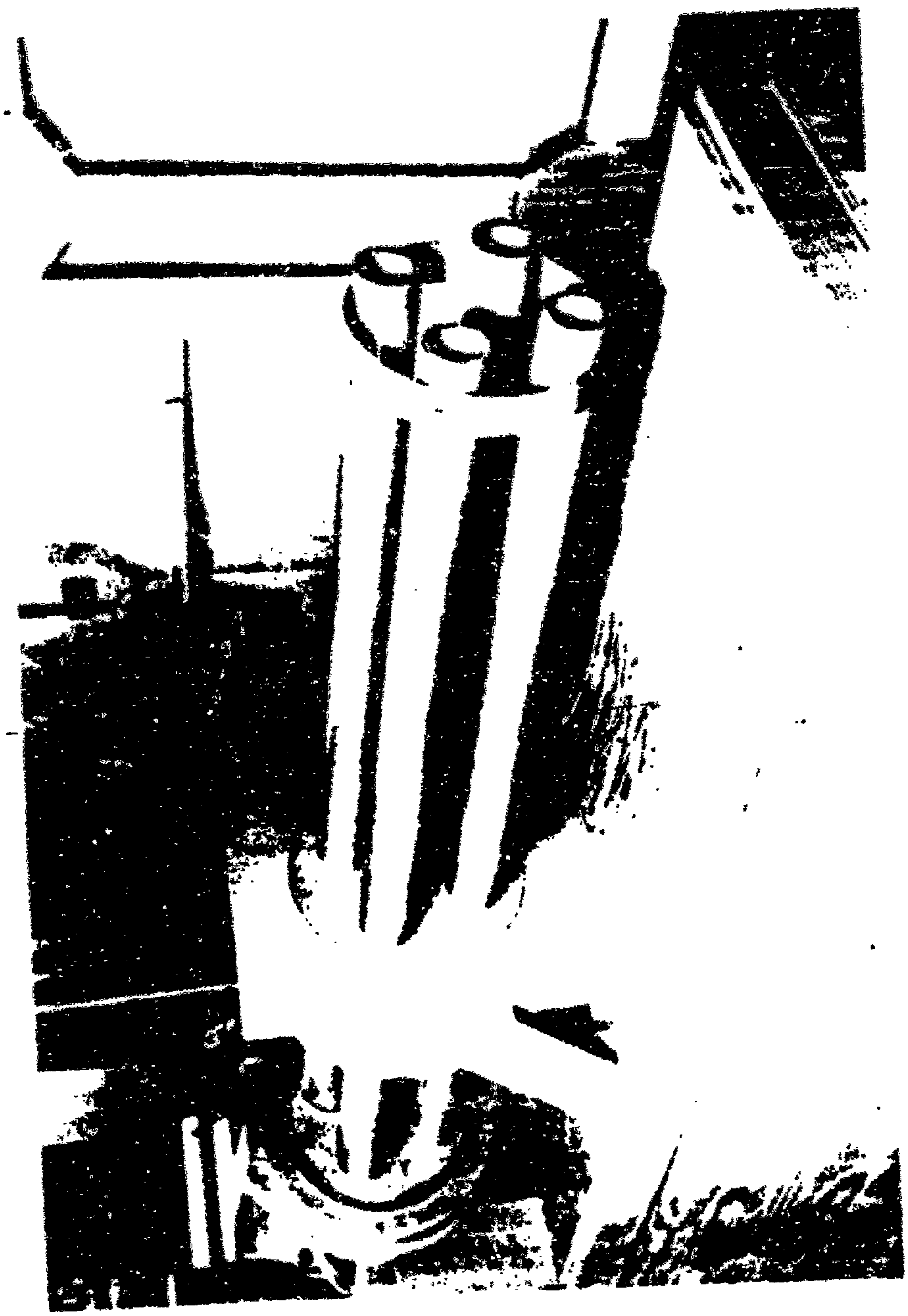


Figure 3-3. 37mm Mockup, Front View

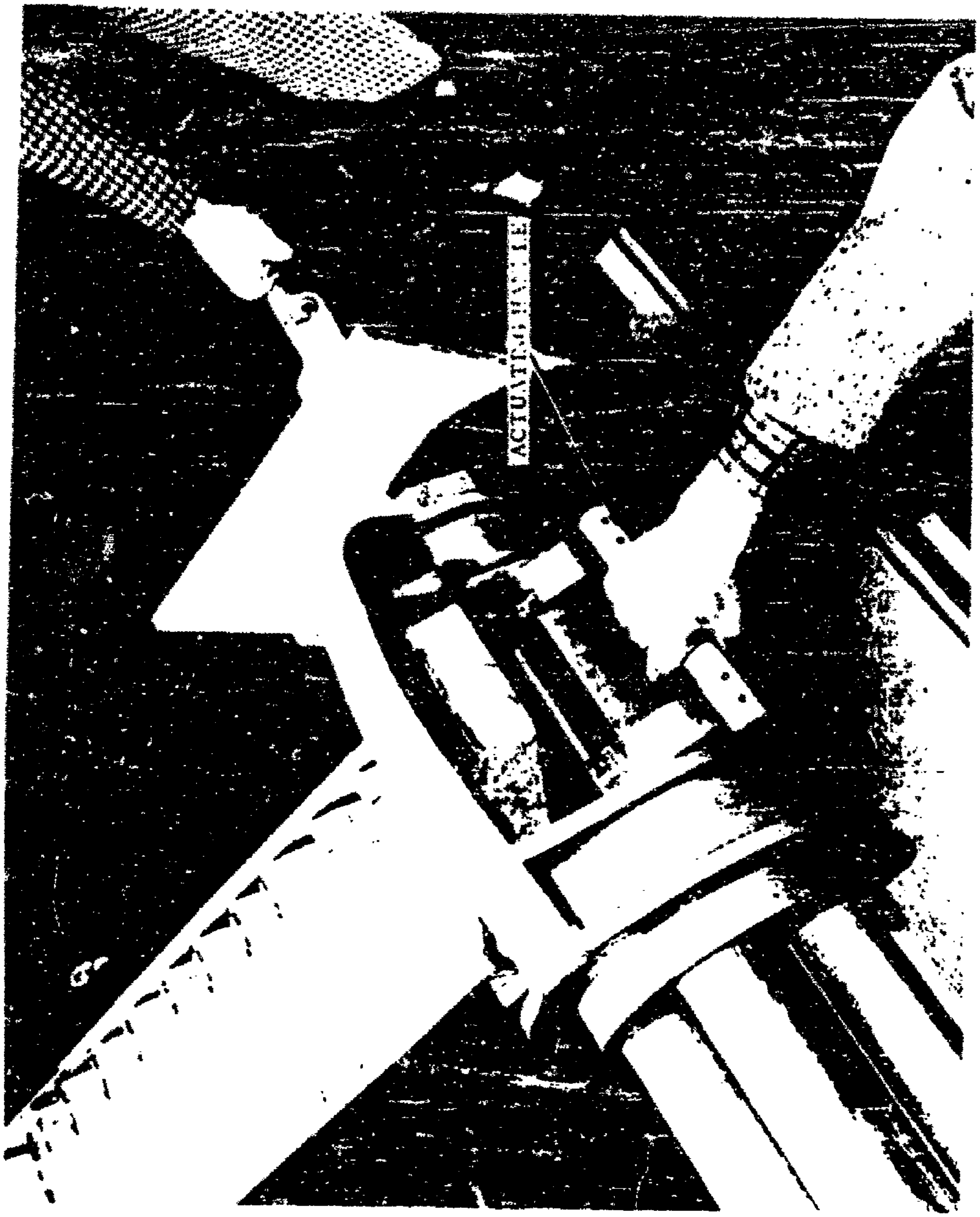


Figure 3-4. First-Round Actuator

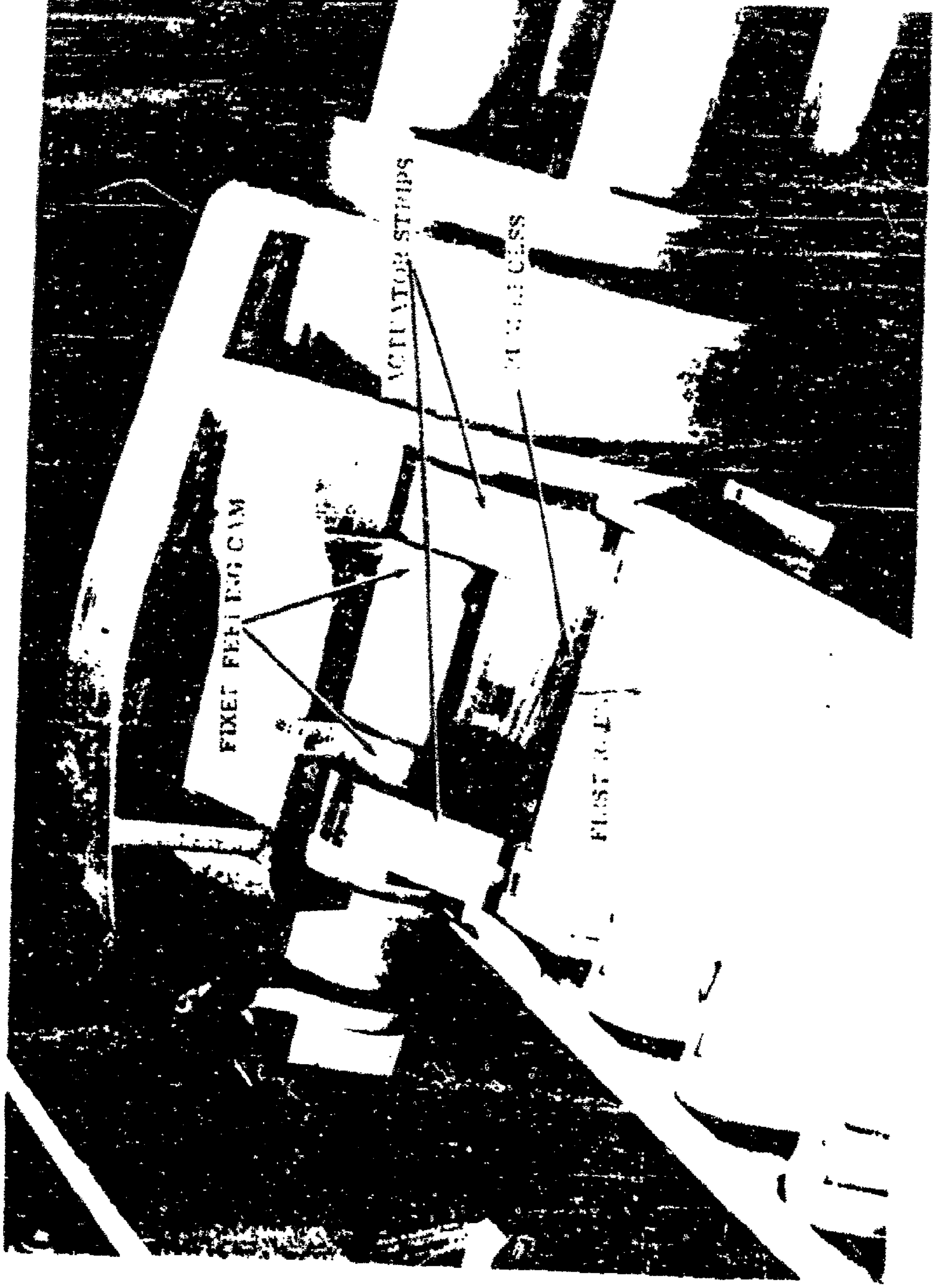


Figure 3-5. First-Round Actuator, Ammunition Side

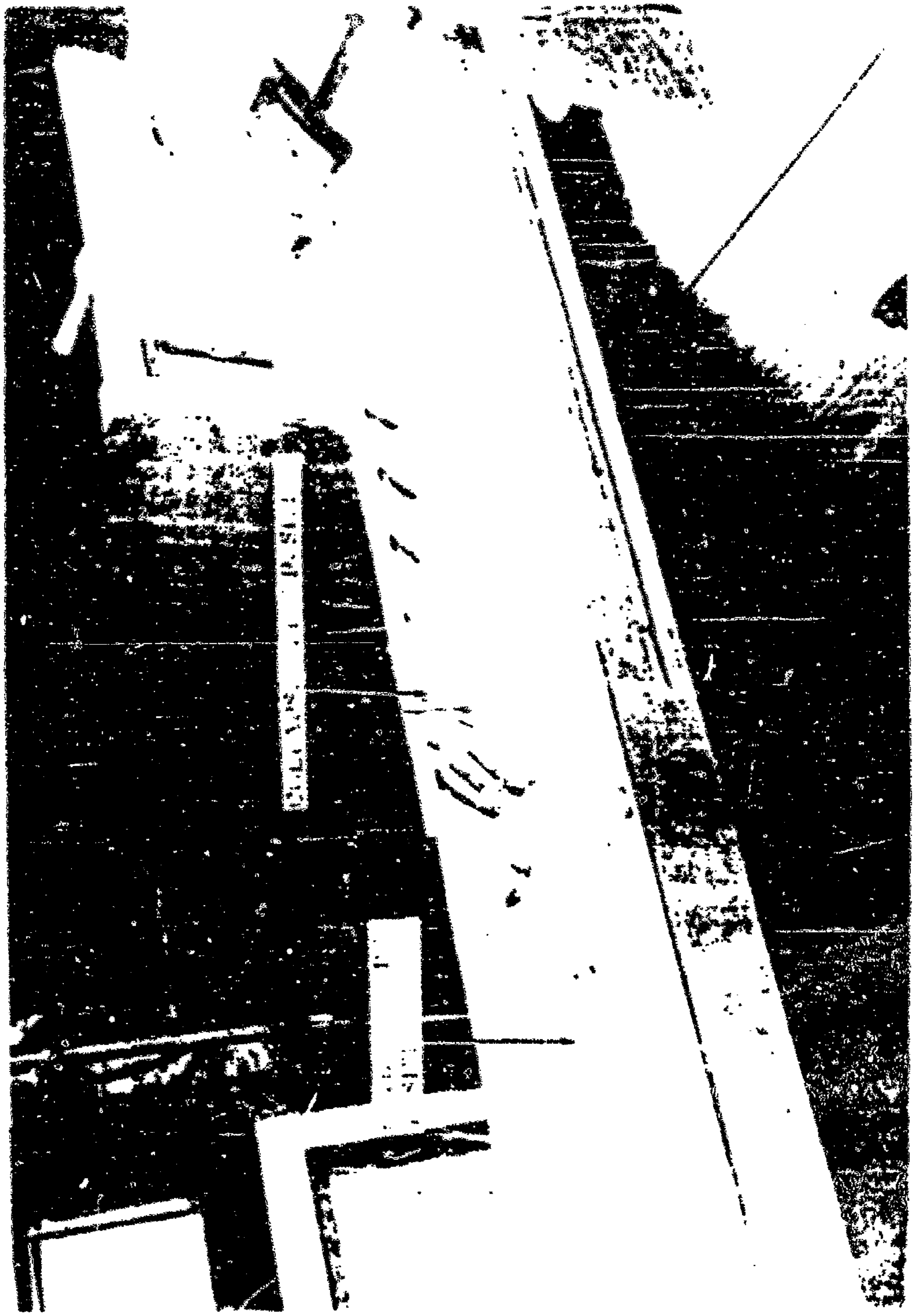


Figure 3-6. Ammunition Belt Closeup

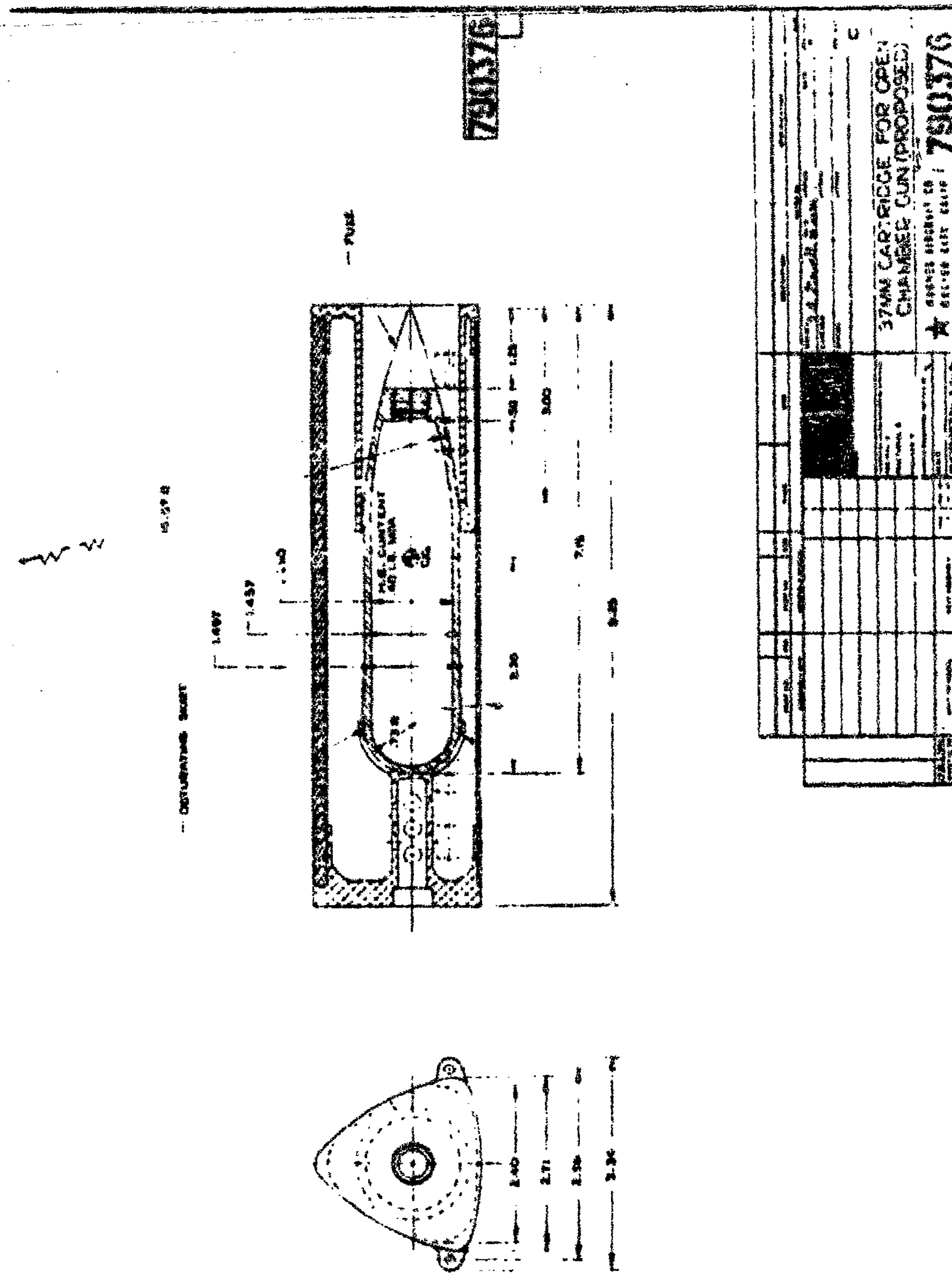
Ammunition

The round is essentially composed of the projectile assembly, cartridge case assembly, propellant, and primer. (See Drawing No. 790376.) The cartridge case is triangular, but may be semi-circular or keystone-shaped. Detailed analysis would be necessary to determine the optimum shape for a given set of conditions. The aluminum cartridge case assembly consists of three principal parts: the case, the front cap, and the rear cap. The case may be made of a continuous extrusion cut to length, the link ears being formed in the extrusion process and the undesired portions being machined off.

The front cap has a thin obturating flange within the main case and a cylindrical sleeve to support and properly index the pre-engraved projectile. The sleeve has an indexing ridge or groove that engages the rotating band and aligns it with the barrel rifling. This sleeve also has radial extensions at the rear for axial centering. The rear closing cap also has an obturating flange; and a support for the rear of the projectile of hemispherical shape. In order to obtain satisfactory ignition, a side vent primer (as illustrated), loaded with a suitable quantity of black powder is recommended for this configuration in preference to an end vent type.

The net volume with the projectile in place is 17.0 cubic inches, which is sufficient, with a double-base powder composition (such as M2) to give the required interior ballistics. (See Section 4, "Interior Ballistics".)

A shot-start device (a shear pin, for example) may be used to hold the projectile in place during the early stages of propellant ignition.


790376
37MM CARTRIDGE FOR OPEN CHAMBER GUN (PROPOSED)
790376

Projectile. A detailed analysis of the projectile is required to ascertain whether it can meet the interior and exterior ballistics requirements and whether it has the volume to contain 0.40 pound MOX or H.E.

Since the geometry of the ammunition indexes it automatically with respect to the barrel rifling, pre-engraved rotating bands may be used. They may be formed as part of the shell body, or made as separate units and attached to the body by welding, brazing, or some similar process. In any event, no band score is necessary within the shell body. This condition is desirable because it leaves the wall intact and permits minimum wall thickness and maximum explosive volume.

A detailed study of the fuse has not been made. It is believed however that a suitable fuse could be designed that would weigh 0.20 pound or less. The fuse should be not only completely bore safe and detonator safe, but it should have both a means for self-destruction and a self-selecting delay to give proper high-order detonation after penetrating light or heavy targets. For high-angle obliquity impact, it is believed that a circumferential front cutting-edge, similar to that of the North American 1.5-inch Maka rocket, would be desirable.

The projectile, including the rotating bands, may be Teflon coated to minimize bore friction. In order to minimize gas leakage past the rear rotating band, a separate, thin, obturating skirt is fastened to the shell body just aft of the band. The skirt, which would be made of soft copper, would expand radially into the rifling.

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Fuse Cavity = $.785 \times .75^2 \times .50 = .22$ cubic inch

Moment of Fuse Cavity = $.22 \times 5.65 = 1.245$ inches^4

$\therefore$ Net Explosive Volume = $6.114 - .220 = 5.894$ cubic inches

Net Explosive Moment = $17.205 - 1.245 = 15.960$ inches^4

CG Explosive = $\frac{15.960}{5.894} = 2.70$ inches from base

Assuming MOX has specific gravity of 2,

Density = .072 pound per cubic inch.

Hence, weight of explosive = $5.894 \times .072 = .423$ pound (MOX)

It is assumed that the fuse weighs .20 pound and its CG is 6.15 inches from the base of the projectile.

TABLE 3-1

WEIGHT AND CG OF PROJECTILE

Part	Weight -- Lb	Moment Arm -- In.	Moment -- Lb In.
Shell Body	.720	2.85	2.050
Explosive Filler	.423	2.70	1.145
Fuse	.200	6.15	1.230
Total	1.343		4.425

CG = $\frac{4.425}{1.343}$ = 3.30 inches from base of shell.

Hence, it may be seen that .40 pound of MOX, or HBX, is a reasonable assumption for the explosive filler.

Stress Analysis of Shell Body. It is now necessary to check the weakest point in the projectile against excessive stress on set-back.

The following is extracted from BRL Technical Note 771, Appendix III

Stress Analysis Calculations:

$$\gamma = \left[X^2 + Y^2 + Z^2 - XY - YZ - ZX\right]^{1/2}$$

γ = Resultant (elastic) stress

X, Y, and Z are the three principal stresses at a point

Tensile stresses are positive, compressive stresses are negative.

Considering the shell wall aft of the rear rotating band, X, Y, and Z are the axial, circumferential, and radial stresses S_A, S_C, and S_R, respectively.

That is, $X = S_A$

$Y = S_C$

$Z = S_R$

$$S_A = \frac{-P \frac{W_S}{W}}{1 - \left(\frac{R_o}{R_1}\right)^2} \quad \text{psi}$$

$$S_C = \frac{P \frac{W_f}{W}\left[1 + \left(\frac{R_1}{R_o}\right)^2\right] - 2P}{1 - \left(\frac{R_o}{R_1}\right)^2} \quad \text{psi}$$

$$S_R = -P \frac{W_f}{W} \left(\frac{R_1}{R_o}\right)^2 \quad \text{psi}$$

P = gas pressure = 38,000 (approx) psi

R_o = inner radius of section = .62 inch

R_1 = outer radius of section = .73 inch

W = weight of shell = 1.35 pounds

W_f = weight of filler = .40 pound

W_S = weight of steel shell + fuze = .60 + .20 = .80 pound

ahead of section being considered

$$X = S_A = \frac{-38{,}000 \times \frac{.80}{1.35}}{1 - \left(\frac{.62}{.73}\right)^2} = -80{,}500 \text{ psi}$$

$$Y = S_C = \frac{38{,}000 \times \frac{.40}{1.35}\left[1 + \left(\frac{.73}{.62}\right)^2\right] - 76{,}000}{1 - \left(\frac{.62}{.73}\right)^2}$$

$$S_C = -175{,}000 \text{ psi}$$

$$Z = S_R = -38{,}000 \times \frac{.40}{1.35}\left(\frac{.73}{.62}\right)^2 = -16{,}000 \text{ psi}$$

From Equation (4) of BRL 771,

$$\bar{\gamma}^2 = \frac{3}{4}(\bar{X} - \bar{Y})^2 + \frac{1}{4}(2\bar{Z} - \bar{X} - \bar{Y})^2$$

where the barred symbols are the unbarred ones divided by f, which is the yield stress of the material, assuming f = 175,000 psi.

$$\bar{X} = \frac{-80{,}500}{175{,}000} = -.485$$

$$\bar{Y} = \frac{-175{,}000}{175{,}000} = -1.0$$

$$\bar{Z} = \frac{-16{,}000}{175{,}000} = -.091$$

$$\bar{\gamma}^2 = \frac{3}{4}(-.485 + 1.0)^2 + \frac{1}{4}\left[-(2 \times .091) + .485 + 1.0\right]^2$$

$$= .199 + .429 = .628$$

$\bar{\gamma} = .792$

$\gamma = .792 \times 175{,}000 = 138{,}000$ psi combined effective stress.

Although this figure is somewhat high it is in the realm of feasibility. A detailed design study would undoubtedly improve this condition.

Moment of Inertia. In order to determine the gyroscopic stability of the projectile in flight, it is necessary to obtain the axial and the transverse moments of inertia. (See Section 5, "Exterior Ballistics".)

Axial

1. Shell Hemisphere

$$I_1 = \frac{1}{3} W_1 R^2 = .33 \times .086 \times .68^2 = .0131 \text{ lb in.}^2$$

2. Shell Cylinder

$$I_2 = \frac{1}{4} W_2 D^2 = .25 \times .433 \times 1.35^2 = .197 \text{ lb in.}^2$$

3. Shell Ogive (Equivalent Cone)

$$I_3 = \frac{3}{10} W_3 r^2 = .3 \times .195 \times .68^2 = .027 \text{ lb in.}^2$$

4. Fuse

$$I_4 = \frac{3}{10} W_4 r^2 = .3 \times .20 \times .50^2 = .015 \text{ lb in.}^2$$

5. Explosive, Hemisphere

$$I_5 = \frac{1}{5} W_5 r^2 = .20 \times .059 \times .63^2 = .0047 \text{ lb in.}^2$$

6. Explosive, Cylinder

$$I_6 = \frac{W_6 D^2}{8} = .302 \times 1.25^2 \times .125 = .0588 \text{ lb in.}^2$$

7. Explosive, Ogive (Equivalent Cone)

$$I_7 = .3\, W_7 r^2 = .3 \times .085 \times .63^2 = .0101 \text{ lb in.}^2$$

Total Axial Moment of Inertia = $\sum_{1-7} I$ = .3257 lb in.^{2}

Transverse Moment of Inertia about CG

1. Shell Hemisphere h = distance of centroid to CG of shell

$$T_1 = I_1 + W_1 h_1^2 = .0131 + .086 \times 2.57^2 = .5831 \text{ lb in.}^2$$

2. Shell Cylinder

$$T_2 = \frac{W_2}{12}(6R^2 + L^2) + W_2 h_2^2$$

$$= \frac{.433}{12}(6 \times .68^2 + 3.42^2) + .433 \times .86^2 = .8420 \text{ lb in.}^2$$

3. Shell Ogive (Equivalent Cone)

$$T_3 = \frac{3W_3}{20}(r^2 + \frac{\ell^2}{4}) + W_3 h_3^2$$

$$= \frac{3 \times .195}{20}(.68^2 + \frac{2.00^2}{4}) + .195 \times 1.498^2 = .4826 \text{ lb in.}^2$$

4. Fuze

$$T_4 = \frac{3W_4}{20}(r^2 + \frac{\ell^2}{4}) + W_4 h_4^2$$

$$= \frac{3 \times .20}{20}(.50^2 + \frac{1.25^2}{4}) + .20 \times 2.85^2 = 1.644 \text{ lb in.}^2$$

5. Explosive, Hemisphere

$$T_5 = I_5 + W_5 h_5^2 = .0047 + .059 \times 2.57^2 = .3947 \text{ lb in.}^2$$

6. Explosive, Cylinder

$$T_6 = W_6 (\frac{D^2}{16} + \frac{\ell^2}{12} + h_5)$$

$$= .302 (\frac{.625^2}{16} + \frac{3.42^2}{12} + .86) = .565 \text{ lb in.}^2$$

7. Explosive, Ogive (Equivalent Cone)

$$I_7 = \frac{3W_7}{20}\left(r^2 + \frac{L^2}{4}\right) + W_7 h_7^2$$

$$= \frac{3 \times .085}{20}\left(.625^2 + \frac{2.00^2}{4}\right) + .085 \times 1.498^2 = .2097 \text{ lb in.}^2$$

Total Transverse Moment of Inertia about CG

$$I = \sum_{1-7} I = 4.7211 \text{ lb in.}^2$$

In Section 5, "Exterior Ballistics", it is shown that with the parameters determined previously -- rate of spin, moments of inertia, location of center of gravity, etc. -- the projectile is stable at sea level.

Complete Round Weight

Cartridge Case.

1. Body — Area = Average thickness times perimeter

Cross-sectional area of metal = .05 x 7.5 = .375 sq in.

Volume = .375 x 8.75 = 3.27 cubic inches

Link ears = .10 x 2 = .20 cubic inch

2. Front Cap

Front closing section = 2.7 x 2.5 x .50 x .18 = .60 cubic inch

Cylinder = .785 $(1.70^2 - 1.5^2)$ x 3.0 = 1.53 cubic inches

Cylinder Supports = .50 x .12 x .75 x 3 = .135 cubic inch

3. Rear Cap

= 2.7 x 2.5 x .50 x .30 = 1.01 cubic inches

Primer = .785 $(.68^2 - .50^2)$ x 2 = .33 cubic inch

Projectile Support = .20 cubic inch

Total = 6.28 cubic inches = .628 pound.

Hinges = 1.0 x .06 x 2.[illegible] x 2 = .33 cubic inches

.33 x .10 = .033 pound per round

Total weight per round

Case Assembly	= .628
Hinge	= .033
Projectile	= 1.350
Propellant	= .460
	2.471 pounds

It may be assumed that 2.50 pounds per round is sufficiently close.

Stress Analysis of Gun

Although a detailed stress analysis of the gun is not within the scope of this study, it is advisable to conduct a comprehensive analysis of the barrel. It is also important to study the critical sections of the drum and breech in order to determine approximate weights and power requirements.

Barrel. The following barrel analysis is based on the method outlined in Reference "s":

$$\frac{S_{ET}}{P_1} = \frac{\sqrt{3U^4 + 1}}{U^2 - 1}$$

S_{ET} = Resultant stress at inner surface -- psi

P_1 = Inner pressure -- psi

$U = \frac{r_o}{r_1}$ = wall ratio

r_o = Outer radius of tube -- inches

r_1 = Radius of bore -- inches

According to Reference "P" a factor of safety of 1.3, and a yield strength of 130,000 psi is taken for the 30mm T-168 gun barrel. This factor is used to compensate for loss of gun material strength at elevated temperatures and to allow for stress concentrations in discontinuous surfaces, such as barrel locks and interrupted threads.

Since it is believed that the major portion of the barrel wall will not have time to reach a high temperature while firing, a factor of safety of 1.2 is felt to be ample. The yield strength of 130,000 psi may be raised considerably by cold working or by auto-frettage methods, as well as by improved materials and heat treatment. Hence,

$$S_{ET} = 130{,}000 \text{ psi}$$

$$P_1 = 1.2P \text{ psi}$$

P is the unit powder pressure in the bore at any position of the projectile.

For P see Section 4, "Interior Ballistics".

$r_1 = .749$ inch = bore radius.

Let

$$A = \frac{\sqrt{3U^4 + 1}}{U^2 - 1} \qquad \text{from} \quad A = \frac{S_{ET}}{P_1}$$

$$A^2 = \frac{3U^4 + 1}{U^4 - 2U^2 + 1}$$

Collecting terms:

$$(A^2 - 3)U^4 - 2A^2U^2 + (A^2 - 1) = 0.$$

Hence:

$$U^2 = \frac{2A^2 \pm \sqrt{4A^4 - 4(A^2 - 3)(A^2 - 1)}}{2(A^2 - 3)}$$

TABLE 3-2

BARREL STRESS ANALYSIS

Station	Distance from Rear -- Inches	P -- Lb/In.^{2} (From Sec. 4)	$P_1 = 1.2P$	$A = \frac{130,000}{P_1}$
1	3.70	37,800	45,400	2.86
2	12.10	30,700	36,900	3.52
3	18.40	26,300	31,500	4.12
4	26.95	22,000	26,500	4.90
5	38.20	17,900	21,500	6.04
6	47.70	14,500	17,400	7.46
7	56.83	11,900	14,300	9.08

TABLE 3-3

BARREL STRESS ANALYSIS (CONT'D)

Station	A	A^2	A^4	$2A^2$	$4A^4$	$4(A^2-3)(A^2-1)$	U^2
1	2.86	8.18	66.91	16.36	267.64	148.6	2.64
2	3.52	12.39	153.5	24.78	614.00	426	2.05
3	4.12	16.97	287.9	33.94	1,151.6	891	1.79
4	4.90	24.01	576.5	48.02	2,306.	1,935	1.60
5	6.04	36.48	1330.8	72.96	5,323.2	4,750	1.445
6	7.46	55.65	3096.9	111.30	12,387.6	11,500	1.34
7	9.08	82.45	6798.0	164.90	27,192.	25,950	1.26

TABLE 3-4

BARREL ANALYSIS (CONCLUDED)

Station	Distance from Rear--Inches	U	r Inches	Wall Thick-ness--Inches	OD -- Inches Theoretical	Actual
1	3.70	1.62	1.215	.466	2.43	2.50
2	12.10	1.43	1.070	.321	2.14	2.24
3	18.40	1.34	1.005	.256	2.01	2.08
4	26.95	1.265	.945	.196	1.89	1.90
5	38.20	1.20	.900	.151	1.80	1.90
6	47.70	1.16	.868	.119	1.736	1.90
7	56.83	1.12	.840	.091	1.68	1.90

It may be noted that the barrel sections have been increased beyond the computed strength requirements in order to provide bearing surfaces and greater stiffness.

Drum and Breech. Figures 3-7 and 3-8 outline a preliminary stress analysis of the drum and breech respectively.

In the case of the drum particularly, the stresses appear rather high. This condition can be improved in actual design by providing a larger radius at the bottom of the recess so that the volume of the case would remain the same but the critical section of the drum would be increased since the recess would be shallower.

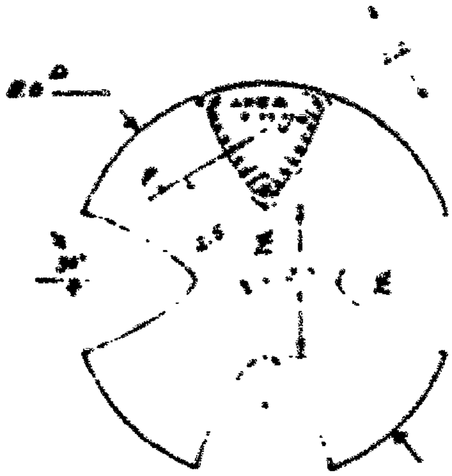
As an example, if the critical section of the drum were to be increased to 3.5 inches by making the recess 0.25 inch shallower, the following would be the approximate stress:

PREPARED BY [illegible] 1-30-[illegible]
CHECKED BY [illegible] 1-31-[illegible]

STRESS ANALYSIS

THE FOLLOWING IS A PRELIMINARY STRESS ANALYSIS OF TWO OF THE MAIN COMPONENTS OF THE 37 MM OPEN CHAMBER GUN; THE DRUM AND THE SHEATH

DRUM



THE CRITICAL CONDITION FOR THE DESIGN OF THE DRUM IS THAT OF A SINGLE EXPLOSION IN EITHER THE UPPER OR LOWER CHAMBERS. THIS UNSYMMETRICAL CONDITION PRODUCES BENDING & TENSION ACROSS A SECTION THROUGH THE ℄

NOTE THAT FOR THIS ANALYSIS ACTUAL PRESSURES ARE COMPARED WITH THE ULTIMATE STRENGTH OF THE MATERIAL. A MORE RIGOROUS ANALYSIS EMPLOYING PLASTIC REDISTRIBUTION WILL SHOW LOWER STRESSES AND THUS PERMIT THE [illegible] TO BE CAPTURED.

P = PRESSURE × INTERNAL WIDTH = (38000)(2.2) = 83600 ppi

$M_{℄} = 2.5\,P = 2.5(83600) = 209000$ in-lb/in

$f_b = \frac{6(209000)}{(3)^2} = 139000$ psi

$f_t = \frac{83600(\cos 30°)}{3} = 24000$ psi

$\overline{163000}$ psi

$F_{tu} = 200000$ psi
4130 STEEL

$M.S. = \frac{200000}{163000} - 1 = \underline{.23}$

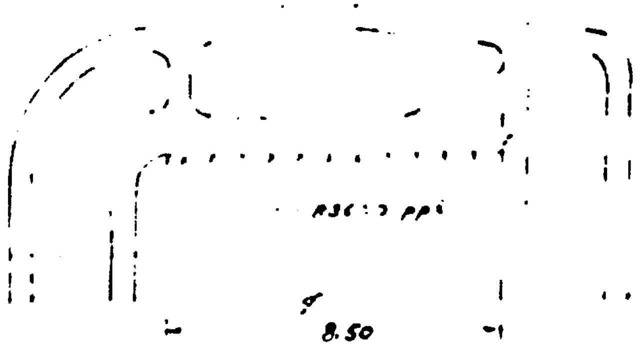
Figure 3-7. Stress Analysis, Drum

PREPARED BY: [illegible] [illegible]
CHECKED BY: [illegible] 1-31-59

STRESS ANALYSIS

BREECH

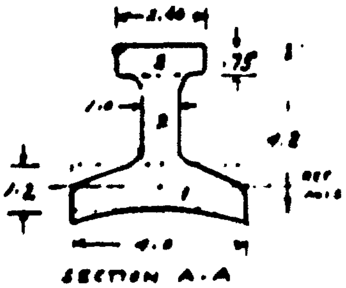
THE BREECH [illegible] [illegible] OF THE [illegible] PRESSURE [illegible] BY THE EXPOSED SURFACE OF THE [illegible] CARTRIDGE. [illegible] CHAMBERS FIRE SIMULTANEOUSLY.



INTERNAL [illegible] CARTRIDGE = 2.2 [illegible]
w = 2.2 × 38000 = 83600 ppi.

ASSUMING THE HORIZONTAL PORTIONS OF THE BREECH TO BE A SIMPLY SUPPORTED BEAM, THE MAXIMUM BENDING MOMENT IS [illegible] TO BE:

$$M_{max} = \frac{wl^2}{8} = \frac{83600(8.5)^2}{8} = 755{,}000 \text{ in.}$$



ITEM	A	d	Ad	Ad^2	I_o
1	4.80	0	0	0	.574
2	2.40	1.8	4.32	7.78	1.150
3	1.80	[illegible]	5.77	18.45	.184
Σ	9.00		10.09	26.23	1.808

$$\bar{d} = \frac{10.09}{9.00} = 1.12$$

$$I = 26.23 + 1.808 - (1.12)(10.09) = 16.74 \text{ in}^4$$

$$f_b = \frac{(755{,}000)(2.48)}{16.74} = 112{,}000 \text{ psi}$$

AXIAL LOAD = [illegible] = [illegible] × 3.73 = 142000 lb

$[F_{tu} = 200000 \text{ psi}]$ $\quad f_t = \frac{142000}{9.00} = 15{,}800 \text{ psi}$

$f_b + f_t = 112{,}000 + 15{,}800 = 127800 \text{ psi}$. $\quad M.S. = \frac{200000}{127800} - 1 = \underline{.57}$

Figure 3-8. Stress Analysis, Breech

$$f_b = \frac{([illegible])(208,000)}{(3.5)^2} = 102,000 \text{ psi}$$

$$f_t = \frac{(83,000)(\cos 30^\circ)}{3.5} = 20,500 \text{ psi}$$

Total stress = 122,500 psi.

This is only one of many possibilities that might be explored to improve stress conditions.

The breech stress of 127,800 psi may also be improved by redesign of the critical section. For example, making top flange (3) (see section shown in Figure 3-8) ½ inch thicker would decrease the stress considerably, yet add only approximately 6.7 pounds to the overall weight. The total area, A, of the section would be increased by 0.60 square inch.

The moment of inertia, I, would increase to 20.69 inches^4. Hence,

f_b = 90,000 psi

f_t = 14,800 psi

Total stress = 104,800 psi.

These stress figures are presented only for the purpose of showing that the basic design is reasonable. Future design work should take into account allowable endurance stress levels for a predetermined life as well as plastic redistribution which tends to lower stresses.

Weight Analysis and Moment of Inertia of Rotating Parts

Barrel. The barrel is considered as being made up of four diametral sections:

TABLE 3-5

BARREL WEIGHT

Section	OD--Inches	Length--Inches	Net Area of Section -- Square Inches	Volume -- Cubic Inches
1	2.5	5.00	3.182	15.9
2	Tapers from 2.4 to 2.0 in. Ave = 2.2	17.00	2.072	35.3
3	2.00	4.00	1.413	5.65
4	1.90	30.83	1.111	34.2
	Total			91.05

Weight of barrel = 91.05 x .283 = 25.7 pounds,

or 26.0 pounds in round figures.

Weight of four barrels = 26 x 4 = 104 pounds.

$$I_o \text{ (own axis)} = \frac{W}{8}(D_1^2 + D_2^2) \qquad D_1 = 2.2 \text{ In. Average}$$

$$D_2 = 1.5 \text{ In.}$$

$$= \frac{104}{8}(4.84 + 2.25)$$

$$= 91.5 \text{ lb In.}^2$$

$$I_{(\text{axis of rotation})} = I_o + Wr^2 \qquad r = 2.95 \text{ In.}$$

$$= 91.5 + 104 \times 2.95^2 = 995.5 \text{ lb In.}^2$$

Drum Assembly. The drum assembly is considered as being made up of seven positive volumes and four negative volumes (holes, recesses, etc.). The following tabular form simplifies the volume and moment of inertia computations.

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TABLE 3-6

DRUM VOLUME AND MOMENT OF INERTIA (SOLID)

Section	Description	Dimensions -- Inches	Volume -- Inches^3	Moment of Inertia -- In.^{5}
1	Solid Drum	8 dia x 10 long	$.785 \times 8^2 \times 10 = 502$	$\frac{502 \times 8^2}{8} = 4016$
2	Four Barrel Supports (Solid)	3 dia x 4.5 long	$.785 \times 3^2 \times 4.5 \times 4 = 128$	$\frac{128 \times 3^2}{8} + 128 \times 2.95^2 = 1254$
3	Front Bearing Flange	9 dia x .25 thick x 3 long	$3.14 \times 8.75 \times .25 \times 3 = 20.5$	$\frac{20.5(9.0^2 - 8.5^2)}{8} = 21.5$
4	Front Vertical Flange	9 dia x .50 thick	$.785 \times 9^2 \times .50 = 31.9$	$\frac{31.9 \times 4.5^2}{2} = 325$
5	Inner Barrel Support Flange (2)	2.9 deep x .50 thick x 3 long	$2.9 \times .5 \times 3 \times 2 = 8.7$	$\frac{8.7 \times 2.9^2}{3} = 24.3$
6	Rear Spindle	2.5 dia x 2.25 long	$.785 \times 2.5^2 \times 2.25 = 11.0$	$\frac{11.0 \times 2.5^2}{8} = 8.6$
7	Rear Spindle Nut	3.5 dia x 1.25 long	$.785 \times 3.5^2 \times 1.25 = 12.0$	$\frac{12 \times 3.5^2}{8} = 14.4$
			714.1	5808.2

Total Solid Volume = 714.1 cubic inches

Total Solid I = 5808.2 inches^5

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TABLE 3-7

NEGATIVE VOLUME AND MOMENT OF INERTIA OF DRUM

Section	Description	Dimensions -- Inches	Volume -- Inches^3	I -- Inches^5
8	Cartridge Recess (4)	Approximate by circle 2.5 dia x 9.25 long	$.785 \times 2.5^2 \times 9.25 \times 4 = 181$	$\frac{181 \times 2.5^2}{8} + 181 \times 2.95^2 = 1711$
9	Front Barrel Recess (4)	2.5 dia x 5 long	$.785 \times 2.5^2 \times 5 \times 4 = 98$	$\frac{98 \times 2.5^2}{8} + 98 \times 2.95^2 = 930$
10	Rear Spindle Hole	2.0 dia x 3.5 long	$.785 \times 2^2 \times 3.5 = 11.0$	$\frac{11 \times 2^2}{8} = 5.5$
11	Triangular Lightening Holes in Drum (4)	1.5-inch base x 10 long	$1.5 \times 1.5 \times 10 \times 2 = 45$	$45 \times 2.95^2 = 393$

Total Negative Volume = 335.0 cubic inches

Total Negative I = 3036.5 Inches^5

$$\text{Net Drum Volume} = \begin{array}{r} 714.1 \\ -\underline{335.0} \\ 379.1 \text{ cubic inches} \end{array}$$

Net Drum Assembly Weight = 379.1 x .283 = 107.5 pounds

$$\text{Net I} = \begin{array}{r} 5808.2 \\ -\underline{3036.5} \\ 2771.1 \text{ inches}^5 \times .283 = 785 \text{ lb in.}^2 \end{array}$$

Barrel Drive Support. This unit is analogous to the front barrel support section of the drum, therefore we add sections 2, 3, and 5 of Table 3-6 and subtract section 9 of Table 3-7. Hence,

$$\begin{aligned} \text{Volume of Drive Support} &= 128 + 20.5 + 8.7 - 98 \\ &= 59.2 \text{ cubic inches} \end{aligned}$$

$$\begin{aligned} I &= 1254 + 21.9 + 24.3 - 930 = 370.2 \text{ inches}^5 \\ &= 105 \text{ lb in.}^2 \end{aligned}$$

Total Weight Rotating Parts.

Barrels	-- 104.0 pounds
Drum Assembly	-- 107.5 pounds
Drive Support	-- 16.8 pounds
Total	228.3 pounds

Total Moment of Inertia of Rotating Parts.

Barrels	-- 995.5 lb in.^{2}
Drum Assembly	-- 785.0 lb in.^{2}
Drive Support	-- 105.0 lb in.^{2}
Total	1885.5 lb in.^{2}

or $$\frac{1885.5}{32.17 \times 144} = .4076 \text{ slug ft}^2$$

These figures are used in a subsequent power requirement analysis.

... is considered as being made up of seven sections as follows:

TABLE 3-8

BREECH VOLUME

Section	Description	Dimensions -- Inches	Volume -- Inches^3
1	Rear Spindle Bearing	4.0 OD x 2.5ID x 3.5 long	$.785(4.0^2 - 2.5^2) \times 3.5$ = 26.8
2	Front Drum Bearing	10.75 OD x 9.0ID x 3.5 long	$.785(10.75^2 - 9^2) \times 3.5$ = 93.5
3	Vertical Center Flange (2)	30.85 In.^{2} x 1.0 thick	30.85 x 1.0 x 2 = 61.7
4	Inner Flange (2)	4 wide x 1.2 thick x 13 long	4 x 1.2 x 13 x 2 = 125.0
5	Outer Flange (2)	2.4 wide x .75 thick x 20 long	2.4 x .75 x 20 x 2 = 72.0
6	Rear Brace Flange (4)	2.5 high x 1 wide x .75 thick	2.5 x 1. x .75 x 4 = 7.5
7	Front Brace Flange (2)	11.4 dia x .6 thick x .9 wide	3.14 x 11.4 x .6 x .9 x 2 = 38.5

Total Breech Volume = 425 cubic inches

Total Breech Weight = 425 x .283 = 120 pounds

Complete Gun Weight.

Barrels	-- 104.0
Drum Assembly	-- 107.5
Drive Support	-- 16.8
Breech	-- 120.0
Total	348.3 without power plant

It has been estimated that a short-life, high-speed, gas turbine plus the necessary cartridges or rocket motor gas generators would weigh between 50 and 75 pounds. Hence the total weight with drive is:

$$348.3 + 75 = 423.3, \text{ or, say } 425 \text{ pounds.}$$

Power Requirement Analysis

Since the rotating parts are brought up to speed before firing occurs, the major consideration is the friction force developed by the cartridge case sliding against the breech during firing.

A recent development of Teflon-coated surfaces shows great promise in furnishing low dynamic friction coefficients. Reference "c" outlines some of the work done in this field. A further reduction in the friction coefficient will probably be obtained at the fairly high sliding velocities (approximately 3000 feet per minute) encountered at the breech-cartridge interface. Reference "d" shows that under high unit loads and high sliding velocities, the coefficient of friction is reduced considerably for practically all solid film lubricants. Although insufficient work has been done with Teflon to predict accurately the coefficient of friction at the pressures and velocities considered, best estimates indicate that it would be 0.01 to 0.02. A coefficient of 0.02 is assumed for these calculations.

First, it is necessary to establish the average pressure during the firing interval. This is done by using the interior ballistic characteristics discussed in Section 4. The average pressure is obtained by dividing the sum of the unit impulse increments imparted to the projectile by the total time in the bore.

TABLE 3-9

AVERAGE PRESSURE AND IMPULSE

$P_{psi\ Ave}$	t -- MS	Impulse -- lb MS/In.^{2}
$\frac{2400 + 18,900}{2} = 10,650$	.88	9,380
$\frac{18,900 + 29,800}{2} = 24,350$	.30	7,300
$\frac{29,800 + 35,600}{2} = 32,700$	.21	6,880
$\frac{35,600 + 37,800}{2} = 36,700$	.19	7,000
$\frac{37,800 + 37,000}{2} = 37,400$	.19	7,100
$\frac{37,000 + 34,300}{2} = 35,650$	.19	6,780
$\frac{34,300 + 30,700}{2} = 32,500$	.22	7,160
$\frac{30,700 + 26,300}{2} = 28,500$	.25	7,150
$\frac{26,300 + 22,100}{2} = 24,200$	.29	7,000
$\frac{22,100 + 17,900}{2} = 20,000$	.35	7,000
$\frac{17,900 + 15,100}{2} = 16,500$	.24	3,950
$\frac{15,100 + 11,900}{2} = 13,500$	.332	4,500

Total time = 3.642 milliseconds

Total impulse = 81,200 lb-MS per square inch

$$\text{Ave } P = \frac{81,200}{3.642} = 22,300 \text{ psi}$$

The internal area of the case expanding radially against the breech

$= 18.9$ square inches

Assuming a coefficient of friction of 0.02, the average force against the breech during firing is:

$$F = 22{,}300 \times 18.9 \times .02 = 8400 \text{ pounds}$$

The braking torque on a four-inch radius is:

$$T_b = 8400 \times \frac{4}{12} = 2800 \text{ pound-feet per station}$$

It may be assumed that the direction of rifling is chosen so that the torque reaction will assist the movement of the rotating parts.

The total average force against the projectile is:

$$F = 22{,}300 \times 1.728 = 38{,}500 \text{ pounds}$$

Since the rifling is one turn in 25 calibers, the force component acting against the lands is $F \tan \alpha$, where α is the angle of the developed rifling.

$$\tan \alpha = \frac{\pi}{25}$$

$$\text{Force component} = \frac{38{,}500 \times 3.14}{25} = 4850 \text{ pounds}$$

Hence the torque reaction due to rifling is:

$$T_r = 4850 \times \frac{1.477 \text{ (Ave dia of bore)}}{12}$$

$$T_r = 600 \text{ pound-feet}$$

Net torque to overcome $T = T_f - T_r$

or $T = 2800 - 600 = 2200$ pound feet per station

or $T = 4400$ pound-feet with two stations firing.

In order to fire 180 rounds per second, the gun revolves

$$\frac{180}{8} = 22.5 \text{ revolutions per second}$$

since there are eight shots per revolution with both stations firing, and the average rotational velocity is

$$22.5 \times 2\pi = 141 \text{ radians per second.}$$

The complete gun cyclic time is $\frac{1}{90}$ = .01111 second. The deceleration period t_d = .00364 second (from Section 4). Hence the acceleration period

$$t_a = .01111 - .00364$$
$$= .00746 \text{ second.}$$

I = moment of inertia of rotating parts = .4076 slug-feet^2

Let $\quad \alpha_d$ = deceleration during firing cycle

t_d = deceleration time during firing cycle

α_a = acceleration during idling cycle

t_a = acceleration time during idling cycle

T_a = torque input of power unit

T_d = net braking torque during firing

If $\quad \omega_1$ = velocity before firing

ω_2 = velocity after firing,

average velocity during firing cycle and idling cycle must be the same to maintain a constant average speed of rotation; that is,

Firing Cycle		Idling Cycle
$\frac{\omega_1 + \omega_2}{2}$	=	$\frac{\omega_2 + \omega_1}{2}$

But $\quad \omega_2 = \omega_1 - t_d\alpha_d$ at the end of the firing cycle

$\omega_1 = \omega_2 + t_a\alpha_a = (\omega_1 - t_d\alpha_d) + t_a\alpha_a$ at the end of the idling cycle

Hence in terms of ω_1 the equation becomes

$$\frac{\omega_1 + (\omega_1 - t_d\alpha_d)}{2} = \frac{(\omega_1 - t_d\alpha_d) + (\omega_1 - t_d\alpha_d) + t_a\alpha_a}{2}$$

Collecting terms:

$$t_a\alpha_a = t_d\alpha_d$$

or Loss in velocity = Gain in velocity.

With $T_d = 4400,\qquad t_d = .00364,\qquad t_a = .00746$

$$\frac{(4400 - T_a)}{I} = \alpha_d$$

Hence loss in velocity $= t_d\alpha_d = \dfrac{.00364 \times (4400 - T_a)}{I}$

$$\frac{T_a}{I} = \alpha_a$$

Gain in velocity $= t_a\alpha_a = \dfrac{.00746\ T_a}{I}$

or $\dfrac{.00364(4400 - T_a)}{I} = \dfrac{.00746\ T_a}{I}$

$T_a = 1440$ pound-feet.

Check: Loss in velocity $= \dfrac{.00364(4400 - 1440)}{.4076} = 26.3$ radians/second

Gain in velocity $= \dfrac{.00746 \times 1440}{.4076} = 26.3$ radians/second

HP Requirement $= \dfrac{T_a}{550} \times \omega\ (\text{rad/sec}) = \dfrac{1440 \times 141}{550}$

$= 369$ HP

This power requirement would, of course, be reduced considerably with a lower coefficient of friction or if one station was fired at a time, rather than two simultaneously.

An even greater reduction of power requirement may be anticipated if a self-consuming case is developed, in which case the friction area would be reduced to a minimum. Proper obturating means would have to be developed.

Heating of Barrels and Cook-off

Cook-off. With this weapon configuration, there is no danger of cook-off since there is never a round in firing position after the gun has stopped firing.

The danger of delayed ignition or hang-fires is minimized by the fact that the round is unconfined once the breech position is passed. Therefore, the propellant, unable to burn under pressure, would simply rupture the cartridge case. Recent experience with 20mm ammunition indicates that major damage does not result under these conditions.

Barrel Cooling. It is believed that since firing takes place so rapidly, there will be no time for heat transfer to the exterior of the barrel. Reference "e" contains a great deal of experimental data to bear this out, particularly in Sections 5.5, "Rates of Heating Under Rapid Fire", and 27.3, "Pre-Engraving of Projectiles".

If necessary, additional cooling may be provided by using special cartridges containing chemical cooling agents. Located at intervals in the ammunition belt, the special cartridges would be fired in the same way as an ordinary round. An auxiliary primer or charge would blow the coolant through the barrel.

Recoil Analysis

To determine the energy of free recoil and other parameters, see Section 4, "Interior Ballistics". For this purpose, the velocity of free recoil will be considered while the projectile is in the bore.

$$V_f = \frac{M + \frac{1}{2} C}{W} V$$

where

V_f = velocity of free recoil of recoiling parts -- feet per second

M = weight of projectile -- pounds = 1.35

C = weight of charge -- pounds = .46

W = weight of recoiling parts -- pounds = 375

V = velocity of projectile at any time t in the bore

X = recoil distance in feet

$\Delta X = V_{f(ave)} \times \Delta t$ for any interval

The energy of free recoil $E_f = \frac{W}{2g} V_f^2$ foot-pounds

TABLE 3-10

RECOIL DISTANCE AND E_f

t--MS	Δt--MS	V'/Sec	V_f	$V_{f(ave)}$	$\Delta X(10^{-3})$	$X(10^{-3})$	V_f^2	E_f
.881	.881	270	1.14	.57	.502	.502	1.3	7.56
1.39	.509	810	3.40	2.27	1.15	1.652	11.5	67.0
1.77	.380	1350	5.68	4.54	1.73	3.382	32.0	186.0
2.18	.410	1890	7.95	6.82	2.8	6.182	63.5	370.0
2.43	.25	2160	9.10	8.52	2.14	8.322	82.8	482.0
2.72	.29	2430	10.2	9.65	2.8	11.122	104.0	605.0
3.15	.43	2750	11.6	10.9	4.68	15.802	135.0	786.0
3.425	.275	2890	12.2	11.9	3.27	19.072	149.0	870.0
3.642	[illegible]	3000	12.6	12.4	2.69	21.762	159.0	925.0

Assuming a recoil spring having an initial compression of F_o and a spring constant K of the system, the total energy absorbed is:

$$F_o X + \frac{K}{2} X^2$$

where F_o = pounds, K = pounds/foot, X = feet of recoil.

Hence, the remaining energy at any instant

$$E = E_f - (F_o X + \frac{K}{2} X^2)$$

At the end of recoil these must equal zero.

With an initial load of 10,000 pounds = F_o and K = 80,000 pounds/foot, an approximate recoil of 0.071 foot would be sufficient to absorb the energy of free recoil:

$$F_o X + \frac{K}{2} X^2 = 10,000 \times .0718 + 40,000 \times .0718^2$$
$$= 718 + 207 = 925 \text{ foot-pounds}$$

The total recoil load would be

10,000 + 5600 = 15,600 pounds with .86 inch recoil.

This figure would actually be less than 15,000, even without considering the use of muzzle brakes, because of factors like friction which would bring the energy of free recoil well under 925 foot-pounds.

It is worth while to consider the use of muzzle brakes, which would reduce recoil loads appreciably.

It may be noted that the momentum of only one shot is used in considering recoil because it is intended to utilize the principle of the Navy 20mm Mk 11 gun where one shot is fired initially and two shots are fired just as the gun is in full counterrecoil position. Thus the energy of counterrecoil is balanced by the energy of recoil of one shot, resulting in a net recoil energy of one shot only.

References

a. Springfield Armory Report, "Stress Distribution in Gun Barrels. Stresses and Strains in Tubes Under External and Internal Pressure", by K. W. Maier. Gun, Auto. Cal .60 T-130
Project No. TS3-3047, SA-TR-1-7000.

b. Report No. 23 on T-168 Gun. USMC Contract DA-19-020-Ord-465.

c. Report C-3270-441/52, BuOrd Project No. 461210, "Teflon Coatings on 20mm Gun Parts", 14 October 1952.

d. NACA Technical Note 1578, "Friction of Solid Films on Steel at High Sliding Velocities".

e. Report of NDRC, Div. 1, Volume 1, "Hypervelocity Guns and the Control of Gun Erosion".

4

INTERIOR BALLISTICS

SECTION 4
INTERIOR BALLISTICS

Discussion

The complete interior ballistic solution for the 37mm gun -- based on the methods discussed in NDRC Report A-348 (Reference 1), "Numerical Methods of Solution of the Ordinary Problems of Interior Ballistics" -- is outlined in this section.

The following differences in fixed parameters were assumed in this case:

1. The factor of 1.04, used in connection with m' (modified mass of projectile), was changed to 1.02 because the former figure assumes 4% of the total energy lost in friction, whereas it is believed that 2% would be a more reasonable figure inasmuch as pre-engraved rotating bands and teflon-coated projectiles would be used. Reference 2a indicates the probability of decreased frictional losses, with NH (non-hygroscopic) powders in a lubricated bore at full charge. Hence

$$m' = \frac{1.02\left(m + \frac{C}{3}\right)}{32.17} \quad \text{(Formula 9, page 4 of Ref. 1)}$$

2. Although pre-engraved rotating bands are used, it has been assumed that the starting conditions for burning the powder are the same as those with conventional projectiles; that is, j_o, which is the fraction of the powder burned at the start of motion, is taken to be 0.01, because a shot-start device will be used to obtain uniform ignition and to prevent debulleting. Reference 1 (page 5) notes that there is very little starting pressure with pre-engraved projectiles. Hence, j_o would be equal to zero if no shot-start device were to be used.

3. The powder chosen, FNH-M2, is of double-base composition with a constant burning surface. This composition is preferable to the single-base FNH-M1 because it has a higher impetus factor, F, so that a cartridge case of smaller volume is required to obtain a given muzzle velocity within a specified maximum pressure. This reduction in case volume is particularly advantageous in the open chamber design since it results in reduced bending moments, smaller friction areas, etc. It is believed that the much higher flame temperature of the M2 powder would not be a great detriment since the duration of firing (1.0 second) would be too short to overheat the entire barrel. Reference 2b states: "It takes a matter of seconds for any considerable part of a pulse of heat received at the bore surface to make its appearance at the outer surface."

4. The ballistic results are believed to be conservative with the specified powder charge weight of 0.46 pound because after a few rounds are fired, the interior surface of the barrel would be at a high temperature, which would result in the transfer of less heat from the powder gases to the barrel wall. Hence $\bar{\gamma}$, which is taken as 1.30 in Reference 1, would be lower. This parameter is the pseudoratio of specific heats that is corrected for heat loss.

5. The muzzle pressure of 11,900 psi, while seemingly high, is actually advantageous if a muzzle brake is going to be used to reduce trunnion reaction.

The following calculations are for preliminary design purposes only. They were, therefore, carried through at slide rule precision.

All of the following parameters, symbols, and equations are those used in NDRC Report A-348 (ATI 24647).

List of Symbols

Symbol	Unit	Meaning
A	Square inches	Cross-sectional area of bore
a	Cubic inches per pound	$\eta - \frac{1}{\rho}$
a^o	--	$2.3969 \times 10^4 (\frac{a}{F})$
B	(in./sec) (lb/in.^{2})	Burning constant
C	Pounds	Weight of powder charge
F	Foot-pounds per pound	Impetus of the powder
f	--	Fraction of the web unburned at any instant
f_o	--	Fraction of the web unburned at the start of motion
I_1, I_2, I_3	--	Tabulated functions used in computing time
j_o	--	Fraction of powder burned at the start of motion
L	Inches	Total travel of projectile
L_x	Inches	Travel up to any point
M	Pounds	Weight of projectile
m'	Slug	Modified mass of projectile [$= 1.02 (M + C/3)/32.17$]
N	Pounds	Weight of powder burned up to any instant
P	Pounds per square inch	Space-averaged pressure behind projectile

Symbol	Unit	Meaning
P_b	Pounds per square inch	Pressure at completion of burning
P_p	Pounds per square inch	Maximum pressure
P^o	Pounds per square inch	Pressure function [= a^o P]
P^o_b	Pounds per square inch	Pressure function at completion of burning [= a^o P_b]
P^o_p	Pounds per square inch	Maximum pressure function [= a^o P_p.
q	--	Parameter depending on starting pressure
T	°K	Temperature of powder gas
T_o	°K	Adiabatic flame temperature of powder gas
t	Seconds	Time of any instant in the motion of projectile
t_b	Seconds	Time from start of motion until powder is all burned
u	--	Parameter depending on $\bar{\gamma}$
V	Feet per second	Velocity of projectile at any instant
V_m	Feet per second	Muzzle velocity
v_c	Cubic inches	Volume of powder chamber
W	Inches	Web thickness of powder grain
X	Inches	Effective distance behind projectile
X_o	Inches	Effective length of powder chamber [= v_c/A]
X/X_o	--	Reduced travel. Ratio of volume behind projectile to chamber volume

Symbol	Unit	Meaning
X_b/X_o	--	Reduced travel to where powder is all burned
X_m/X_o	--	Reduced travel to muzzle. Ratio of total volume of gun to chamber volume
x_1	--	$f_o \xi/(1 + f_o \xi)Z_b$
x_2	--	$f_o \psi/(1 - J_o \psi)Z_b$
Z	--	Parameter, reduced velocity for the interval of burning
Z_b	--	Value of Z at instant powder is all burned, called burning function
Z_p	--	Value of Z at time of maximum pressure
a	--	Parameter depending on density of loading
Γ	--	Velocity function
γ	--	Ratio of specific heats
$\bar{\gamma}$	--	Pseudoratio of specific heats corrected for heat loss [taken to be 1.30]
Δ	Grams/cubic centimeter	Density of loading [$= 27.68C/v_c$]
Δ_o	Pounds per cubic inch	Density of loading [$= C/v_c$]
η	cubic inches per pound	Covolume of powder gas
ξ	--	Travel function [$= a\Delta_o/(X/X_o - \eta\Delta_o)$]
ξ_b	--	Travel function evaluated to where powder is all burned
ξ_m	--	Travel function at shot ejection

Symbol	Unit	Meaning
ρ	Pounds per cubic inch	Density of solid powder
φ	—	Density of loading function $[= a(1/\Delta_o - 1/\rho)^{-1}]$

Performance Estimate (See page 14, A-348)

Gun Constants

Bore = 37mm (1.457 inches)

v_o = 17.00 cubic inches (chamber volume)

L = 63.13 inches (total projectile travel)

A = 1.728 square inches (area of bore)

$X_e/X_o = 7.42$

Powder Constants for FNH-M2 (Table XIV, A-348)

F = 384,000 foot-pounds per pound (impetus)

η = 26.95 cubic inches per pound

$\frac{1}{\rho}$ = 16.78 cubic inches per pound

a = 10.17 cubic inches per pound

$a^o = 0.635$

$T_o = 3560°$ K (adiabatic flame temperature)

Loading Constants

k = 1.35 pounds (projectile weight)

Δ = 0.75 grams per cubic centimeter (density of loading)

C = 0.46 pound (powder charge)

P_p = 37,800 pounds per square inch (maximum allowed pressure)

Quantity	Numerical Value
(1) θ (From Table XIII, A-348)	.4454
(2) $P_p^0 = s^0 P_p = .635 \times 37,800$	24,000 lb/in^2
(3) Γ (From Table IV, A-348)	1.6710
(4) $\Delta_o = \Delta/27.68 = .75/27.68$	0.027 lb/in^3
(5) $C = v_c \Delta_o = 17.0 \times .027$	0.46 pound
(6) $m' = 1.02(M + C/3)/32.17 = 1.02(1.35 + .46/3)/32.17$	0.0475 slugs
(7) $\xi_m = a\Delta_o/(Z_m/Z_o - \eta\Delta_o)$ $= 10.17 \times .027/(7.42 - 26.95 \times .027)$	0.0410
(8) $\xi_m^{0.3}$ (From Table XII, A-348)	0.38356
(9) $V_m^2 = \frac{CF}{0.15\, m'} (1 - \Gamma \xi_m^{0.3})$ $= \frac{.46 \times 384,000}{0.15 \times .0475} (1 - 1.671 \times .38356)$	9.0×10^6
(10) V_m = (Muzzle velocity)	3000 ft/sec

In order to ascertain that the powder is completely burned before shot ejection, check that

$$uf_o Z_b < (1 - \Gamma \xi^{0.3})$$

$u = 0.15$ (From page 12, A-348)

$f_o = 0.99$ (From page 12, A-348)

$Z_b = 2.037$ (From Table III, A-348)

$$uf_o Z_b = .15 \times .99 \times 2.037 = .302$$

$$1 - \Gamma \xi_m^{0.3} = 1 - 1.671 \times .38356 = .361$$

Since $.302 < .361$, the powder is all burned before shot ejection.

Complete Solution of Interior Ballistics

A. Period of Burning of Powder. It is assumed that a constant burning surface powder grain is used.

From page 16, A-348

$$(B/W)^2 = A^2 f_o / CFm'Z_b$$

B = burning constant = 0.0005 for M2 powder (Reference 2c)

W = powder grain web (inches).

From pages 4-6 and 4-7,

$A = 1.728$ $F = 384{,}000$ $Z_b = 2.037$

$f_o = .99$ $m' = .0475$ $C = .46$

Hence

$$(\frac{B}{W})^2 = \frac{0.99 \times 1.728^2}{.46 \times 384{,}000 \times .0475 \times 2.037} = 1.74 \times 10^{-4}$$

$$\frac{B}{W} = .0132$$

$$W = \frac{.0005}{.0132} = .0379 \text{ in.}$$

The following additional parameters are necessary to tabulate other functions (page 24, A-348):

$$x_2 = \frac{f_o \phi}{(1 - j_o \phi)Z_b} = \frac{.99 \times .4454}{(1 - .01 \times .4454)2.037} = .218$$

$\phi = .4454$ (from page 4-7)

$j_o = .01$

$q = j_o Z_b / f_o = .01 \times 2.037/.99 = .0206$

The following table contains values of Z as one argument up to $Z_b = 2.037$. x_1 is obtained by interpolating within Table IX, A-348 for q = .02 (page 81, A-348), and $x_2 = .218$

$I_1(Z)$ and $I_2(Z)$ are obtained from Table I, pages 85 and 86 of A-348.

TABLE 4-1

Z	X_1	$X_1 Z_b$	$I_1(Z)$	$I_2(Z)$
0	.21800	.443		
0.2	.18750	.381	.01557	.00100
0.4	.15583	.317	.02088	[illegible]
0.6	.12859	.262	.02494	.00404
0.8	.10547	.214	.02867	.00600
1.0	.08587	.175	.03244	.00832
1.2	.06934	.141	.03646	.01110
1.4	.05546	.113	.04096	.01447
1.6	.04388	.0891	.04617	.01861
1.8	.03435	.0698	.05237	.02377
2.0	.02647	.0538	.05996	.03027
2.037	.02523	.0514	.06161	.03172

$Z_b = 2.037$

From the above table, solve for ξ^{-1} (see page 25, A-348)

$$\xi^{-1} = \frac{(1 - x_1 Z_b)\mu_0}{x_1 Z_b}$$

TABLE 4-2

INTERIOR BALLISTICS DURING BURNING OF POWDER

(1) Z	(2) ξ^{-1}	(3) X/X_c	(4) L_x (Inches)	(5) t (10^{-3}sec)	(6) P (Lb/in^2)	(7) V (Ft/sec)	(8) T °K	(9) N/C
0		1	0	0	2,040	0	3560	.01
.2	1.605	1.169	1.67	.88	18,900	270	3460	.1075
.4	2.127	1.312	3.08	1.18	29,800	540	3360	.2050
.6	2.780	1.493	4.86	1.39	35,600	810	3260	.3025
.8	3.640	1.728	7.16	1.58	37,800	1080	3150	.4000
1.0	4.675	2.013	10.00	1.77	37,000	1350	3040	.4975
1.2	6.030	2.388	13.65	1.96	34,300	1620	2930	.5950
1.4	7.77	2.863	18.40	2.18	30,700	1890	2820	.6925
1.6	10.10	3.503	24.70	2.43	26,300	2160	2720	.7900
1.8	13.25	4.378	33.25	2.72	22,100	2430	2610	.8875
2.0	17.40	5.513	44.50	3.07	17,900	2700	2500	.9850
2.037 (burned)	18.30	5.758	46.80	3.15	17,200	2750	2490	1.000

For solution of columns (3) and (4), see page 4-12

For solution of column (5), see page 4-13

For solution of column (6), see page 4-14

For solution of column (7), see page 4-15

For solution of column (8), see page 4-15

For solution of column (9), see page 4-16

TABLE 4-3

INTERIOR BALLISTICS AFTER BURNING

(1) L_ζ (In.)	(2) X/X_o (In.)	(3) ζ	(4) $\zeta^{0.3}$	(5) $\Gamma\zeta^{0.3}$	(6) $I_3(\Gamma\zeta^{0.3})$	(7) t (10^{-3}sec)	(8) P (Lb/in²)	(9) v (Ft/sec)	(10) T °K
(Burned) 46.80	5.758	.0545	.41775	.699	74.6176	3.150	17,200	2750	2490
48.0	5.875	.0533	.41497	.693	74.5649	3.194	16,700	2770	2470
50.0	6.08	.0514	.41048	.685	74.4927	3.255	16,000	2800	2450
52.0	6.28	.0495	.40586	.678	74.4274	3.310	15,100	2830	2415
54.0	6.47	.0478	.40163	.670	74.3504	3.374	14,500	2860	2390
56.0	6.70	.0460	.39703	.664	74.2897	3.425	13,800	2890	2360
58.0	6.89	.0446	.39337	.657	74.2172	3.485	13,200	2920	2330
60.0	7.10	.0431	.38935	.650	74.1426	3.549	12,600	2950	2310
62.0	7.30	.0418	.38579	.644	74.0753	3.605	12,100	2980	2290
63.13 (Shot ejection)	7.43	.0410	.38356	.640	74.0304	3.642	11,900	3000	2275

For solution of column (2), see page 4-17

For solution of column (3), see page 4-17

For solution of column (4), see Table XII, A-348

For solution of column (6), see Table XI, A-348

For solution of column (7), see page 4-18

For solution of column (8), see page 4-19

For solution of column (9), see page 4-20

For solution of column (10), use $T = \Gamma\zeta^{0.3}\, T_o$ (See 22H page 27, A-348)

$$\Gamma = 1.671 \qquad T_o = 3560^\circ \text{ K}$$

See figures 4-1 and 4-2 for pressure and velocity as functions of travel and time, respectively.

Solut of Columns (3) and (4) -- Table 4-2

$$\frac{X}{X_o} = (\eta + a\,\xi^{-1})\Delta_o \qquad \text{(See 9H page 25, A-348)}$$

$\eta = 26.95 \qquad \Delta_o = .027 \qquad \eta\Delta_o = .728$

$a = 10.17 \qquad a\Delta_o = .275$

or

$$\frac{X}{X_o} = \eta\Delta_o + a\Delta_o\,\xi^{-1} = .728 + .275\,\xi^{-1}$$

Also,

$$L_x = \left(\frac{X}{X_o} - 1\right)\frac{v_c}{\lambda} \qquad \text{(See 10H page 25, A-348)}$$

$$\frac{v_c}{\lambda} = \frac{17}{1.728} = 9.85$$

Hence,

$$L_x = \left(\frac{X}{X_o} - 1\right) 9.85$$

TABLE 4-4

ξ^{-1}	$.275\,\xi^{-1}$	X/X_o	L_x
1.605	.441	1.169	1.67
2.127	.584	1.312	3.08
2.780	.765	1.493	4.86
3.640	1.00	1.728	7.16
4.675	1.285	2.013	10.00
6.030	1.66	2.388	13.65
7.770	2.135	2.863	18.40
10.10	2.775	3.503	24.70
13.25	3.65	4.378	33.25
17.40	4.785	5.513	44.50
18.30	5.030	5.758	46.80

Solution of Column (5) -- Table 4-_, page 4-10

$$t = \frac{a\, j_o}{12F\, q^2 \cdot q^{0.3} (B/W)} \left[x_2^{-1}\, I_1(Z) - I_2(Z) \right]$$

For the above, see formula 11H page __, A-348.

$a = 10.17$ $F = 384{,}000$ $j_o = .01$

$q = .0206$ (see page 4-8) $q^2 = .000425$

$q^{.3} = .3120$ $x_2 = .218$ (see page 4-8)

$B/W = .0132$ (see page 4-8)

$$t = \frac{10.17 \times .01}{12 \times 384{,}000 \times .000425 \times .312 \times .0132} \left[\frac{I_1(Z)}{.218} - I_2(Z) \right]$$

or,

$$t = .0126 \left[\frac{I_1(Z)}{.218} - I_2(Z) \right]$$

From Table 4-1 on page 4-9, use $I_1(Z)$ and $I_2(Z)$

TABLE 4-5

SOLUTION FOR t

Z	$\frac{I_1(Z)}{.218}$	$\frac{I_1(Z)}{.218} - I_2(Z)$	t (Sec)
.2	.071	.070	.000881
.4	.096	.0936	.00118
.6	.114	.110	.00139
.8	.1315	.1255	.00158
1.0	.1485	.1402	.00177
1.2	.1665	.1554	.00196
1.4	.1875	.1730	.00218
1.6	.2120	.1934	.00243
1.8	.240	.2162	.00272
2.0	.275	.2447	.00307
2.037	.2825	.2508	.00315

Solution of Column (6) -- Table 4-2, Page 4-10

From formula 12M page 25, A-348

$$P = \frac{2.8763 \times 10^5 \, (q + Z - uZ^2)x_1}{a^o \, (1 - x_1 \, Z)}$$

$q = .0206$ (see page 4-8) $\quad a^o = 0.635 \quad u = .15$

$$P = \frac{2.8763 \times 10^5(.0206 + Z - .15Z^2)x_1}{.635(1 - x_1 \, Z)}$$

Let $\quad d = (.0206 + Z - .15Z^2)$

Hence
$$P = \frac{2.8763 \times 10^5 \times d \times x_1}{.635(1 - x_1 Z)}$$

TABLE 4-6

SOLUTION FOR P

Z	x_1 (p. 4-9)	x_1Z	uZ^2	d	$1-x_1Z$	$a^o(1-x_1Z)$	P
0	.21800	0	0	.0206	1.00	.635	2,040
.2	.18750	.0375	.006	.2146	.9625	.611	18,900
.4	.15583	.0623	.024	.3966	.9377	.595	29,800
.6	.12859	.0770	.054	.5656	.9230	.586	35,600
.8	.10547	.0842	.096	.7246	.9158	.581	37,800
1.0	.08587	.0859	.150	.8706	.9141	.580	37,000
1.2	.06934	.0834	.215	1.0056	.9166	.582	34,300
1.4	.05546	.0776	.294	1.1266	.9224	.585	30,700
1.6	.04388	.0700	.384	1.2366	.9300	.590	26,300
1.8	.03435	.0618	.485	1.3356	.9382	.596	22,100
2.0	.02647	.0529	.600	1.4206	.9471	.600	17,900
2.037	.02523	.0514	.622	1.4356	.9486	.602	17,200

Solution of Column (7) -- Table 4-2, Page 4-10

From Formula 13H page 25, A-348

$$V = Z\sqrt{\frac{C\ F\ f_o}{m'\ Z_b}}$$

$C = .46$ $\qquad Z_b = 2.037$

$F = 384{,}000$ $\qquad f_o = .99$

$m' = .0475$ (See page 4-7)

$V = 13502$

Solution of Column (8) -- Table 4-2, Page 4-10

From Formula 14H page 25, A-348

$$T = T_o\left[\frac{1 - 0.15Z^2}{q + Z}\right] = T_o\ [1 - h]$$

$q = .0206,$ $\qquad T_o = 3560^\circ$ K, $\qquad$ let $h = \frac{.15Z^2}{q + Z}$

TABLE 4-7

SOLUTION FOR T

Z	$.15Z^2$	q + Z	h	1 - h	T (°K)
0	0		0	1.0	3560
.2	.006	.2206	.027	.973	3460
.4	.024	.4206	.057	.943	3360
.6	.054	.6206	.087	.913	3260
.8	.096	.8206	.117	.883	3150
1.0	.150	1.0206	.147	.853	3040
1.2	.215	1.2206	.176	.824	2930
1.4	.294	1.4206	.207	.793	2820
1.6	.384	1.6206	.237	.763	2720
1.8	.485	1.8206	.267	.733	2610
2.0	.600	2.0206	.297	.703	2500
2.037	.622	2.0576	.302	.698	2490

Solution of Column (9) -- Table 4-2, Page 4-10

From formula 15N page 25, A-348

$$\frac{N}{C} = j_o + f_o \, Z/Z_b$$

$$j_o = .01, \qquad f_o = .99 \qquad Z_b = 2.037$$

Hence

$$\frac{N}{C} = .01 + .4875\, Z$$

TABLE 4-8

SOLUTION FOR N/C

Z	.4875 Z	N/C
0	0	.010
.2	.0975	.1075
.4	.1950	.2050
.6	.2925	.3025
.8	.3900	.400
1.0	.4875	.4975
1.2	.5850	.5950
1.4	.6825	.6925
1.6	.7800	.7900
1.8	.8775	.8875
2.0	.9750	.9850
2.037	.990	1.00

Solution of Column (2) -- Table 4-3, Page 4-11

From Formula (10H) page 25, A-348

$$L_x = \left(\frac{X}{X_c} - 1\right)\frac{v_c}{A}$$

$$\frac{X}{X_o} = \frac{L_x}{\frac{v_c}{A} + 1} \qquad\qquad \frac{v_c}{A} = 9.85$$

Hence

$$\frac{X}{X_o} = \frac{L_x}{9.85} + 1$$

Solution of Column (3) -- Table 4-3, Page 4-11

From Formula (61) page 35, A-348

$$\xi = \frac{a\Delta_o}{\frac{X}{X_o} - \eta\Delta_o} = \frac{.275}{\frac{X}{X_o} - .728}$$

$a\Delta_o = .275$ (page 4-12) $\qquad\qquad \eta\Delta_o = .728$ (page 4-12)

TABLE 4-9

SOLUTION FOR ξ

L_x	$\frac{X}{X_o}$	$\frac{X}{X_o} - .728$	ξ
46.80	5.758	5.030	.0545
48.0	5.875	5.147	.0533
50.	6.08	5.352	.0514
52.	6.28	5.552	.0495
54.	6.47	5.742	.0478
56.	6.70	5.972	.0460
58.	6.89	6.162	.0446
60.	7.10	6.372	.0431
62.	7.30	6.572	.0418
63.13	7.43	6.702	.0410

Solution of Column (7) -- Table 4-3, Page 4-11

From Formula (.38) page 27, A-348

$$t = t_b + \frac{a\Gamma^{1/0.3}}{12A}\sqrt{\frac{Cm'}{0.6P}}\left[I_3(\Gamma\xi_b^{.3}) - I_3(\Gamma\xi^{.3})\right]$$

$t_b = .00315$ (page 4-11)	$m' = .0475$	$a = 10.17$
$P = 384{,}000$	$A = 1.728$	$\Gamma = 1.6710$ (p. 4-7)
$C = .46$	$\Gamma^{1/.3} = \Gamma^{10/3} = 5.537$	$I_3 = (\Gamma\xi_b^{.3}) = 74.6176$

Obtain Column (3) below by interpolating within Table XI, A-348,

$$t = .00315 + \frac{10.17 \times 5.537}{12 \times 1.728}\sqrt{\frac{.46 \times .0475}{.6 \times 384{,}000}}\left[74.6176 - I_3(\Gamma\xi^{.3})\right]$$

$$= .00315 + .000838\left[74.6176 - I_3(\Gamma\xi^{.3})\right]$$

TABLE 4-10

SOLUTION FOR t

L_x	$(\Gamma\xi^{.3})$*	$I_3(\Gamma\xi^{.3})$	$74.6176 - I_3(\Gamma\xi^{.3})$	x .000838	t (Seconds)
46.80	.699	74.6176	0	0	.003150
48.0	.693	74.5649	.0527	.000044	.003194
50.	.685	74.4927	.1249	.000105	.003255
52.	.678	74.4274	.1902	.000160	.003310
54.	.670	74.3504	.2672	.000224	.003374
56.	.664	74.2897	.3279	.000275	.003425
58.	.657	74.2172	.4004	.000335	.003485
60.	.650	74.1426	.4750	.000399	.003549
62.	.644	74.0753	.5423	.000455	.003605
63.13	.640	74.0304	.5872	.000492	.003642

* $\Gamma\xi^{.3}$ from Column (5), Table 4-3

Solution of Column (8) -- Table 4-3, Page 4-11

From Formula (20H) page 27, A-348

$$P = 2.8763 \times 10^5 \, \Gamma \, \xi^{1.3} / a^0$$

$$\Gamma = 1.6710 \qquad \text{(See page 4-7)}$$

$$a^0 = .635$$

$$P = \frac{2.8763 \times 10^5 \times 1.671}{.635} \, \xi^{1.3}$$

$$= 755{,}000 \; \xi^{1.3}$$

Solve $\xi^{1.3}$ from $\text{Log } \xi^{1.3} = \text{Log } \xi + \text{Log } \xi^{.3}$

TABLE 4-11

SOLUTION FOR P

L_x	ξ	$\xi^{.3}$	Log ξ	Log $\xi^{.3}$	Log $\xi^{1.3}$	$\xi^{1.3}$	P
48.0	.0533	.41497	$\bar{2}$.726727	$\bar{1}$.618021	$\bar{2}$.344745	.02212	16,700
50.0	.0514	.41048	$\bar{2}$.710963	$\bar{1}$.613292	$\bar{2}$.324255	.02110	16,000
52.0	.0495	.40586	$\bar{2}$.694605	$\bar{1}$.608376	$\bar{2}$.302961	.02009	15,100
54.0	.0478	.40163	$\bar{2}$.679428	$\bar{1}$.603826	$\bar{2}$.283254	.01920	14,500
56.0	.0460	.39703	$\bar{2}$.662758	$\bar{1}$.598824	$\bar{2}$.261582	.01826	13,800
58.0	.0446	.39337	$\bar{2}$.649335	$\bar{1}$.594802	$\bar{2}$.244137	.01754	13,200
60.0	.0431	.38935	$\bar{2}$.634477	$\bar{1}$.590339	$\bar{2}$.224816	.01678	12,600
62.0	.0418	.38579	$\bar{2}$.621176	$\bar{1}$.586350	$\bar{2}$.207526	.01613	12,100
63.13	.041	.38356	$\bar{2}$.612784	$\bar{1}$.583833	$\bar{2}$.196617	.01573	11,900

Solution of Column (9) -- Table 4-3, Page 4-11

From Formula (21M) page 27, A-348

$$V^2 = \frac{2CF}{0.3m'} (1 - \Gamma \zeta^{0.3})$$

$C = .46$ $\Gamma = 1.671$

$F = 384{,}000$ $m' = .0475$

$$V^2 = \frac{2 \times .46 \times 384{,}000}{.3 \times .0475} (1 - \Gamma \zeta^{.3})$$

$$= 24.8 \times 10^6 (1 - \Gamma \zeta^{.3})$$

TABLE 4-12

SOLUTION FOR V

L	$\Gamma \zeta^{.3}$	$1 - \Gamma \zeta^{.3}$	V^2	V
48.0	.693	.307	7.62×10^6	2770
50.0	.685	.315	7.81×10^6	2800
52.0	.678	.322	8.00×10^6	2830
54.0	.670	.330	8.19×10^6	2860
56.0	.664	.336	8.35×10^6	2890
58.0	.657	.343	8.50×10^6	2920
60.0	.650	.350	8.68×10^6	2950
62.0	.644	.356	8.87×10^6	2980
63.13	.640	.360	9.00×10^6	3000

References

1. NDRC Report A-348. Interior Ballistics VII Numerical Methods of Solution of the Ordinary Problems of Interior Ballistics. (ATI-24647).

The references used are cited directly in the text as required.

2. NDRC Report of Div. 1 Volume 1.

Hypervelocity Guns and the Control of Gun Erosion.

a. Table 2, page 143 -- Summary of Results Pertaining to Bore Friction from Firing Three Inch Gun.

b. Paragraph 5.5.1, page 116.

c. Table 3, page 62 -- Approximate Values of Burning Rate Constant, Typical Powders.

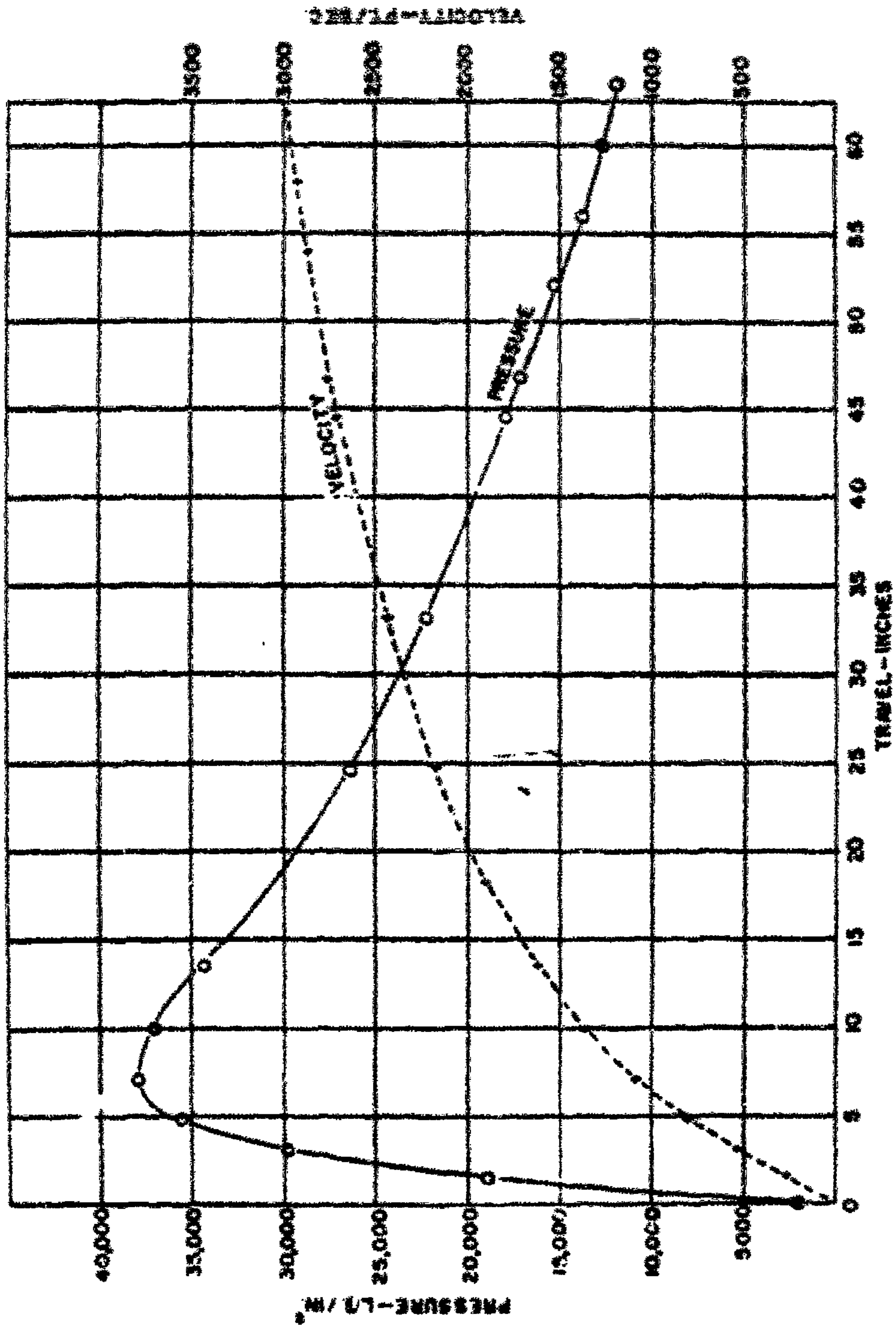

Figure 4-1. Pressure and Velocity vs Projectile Travel

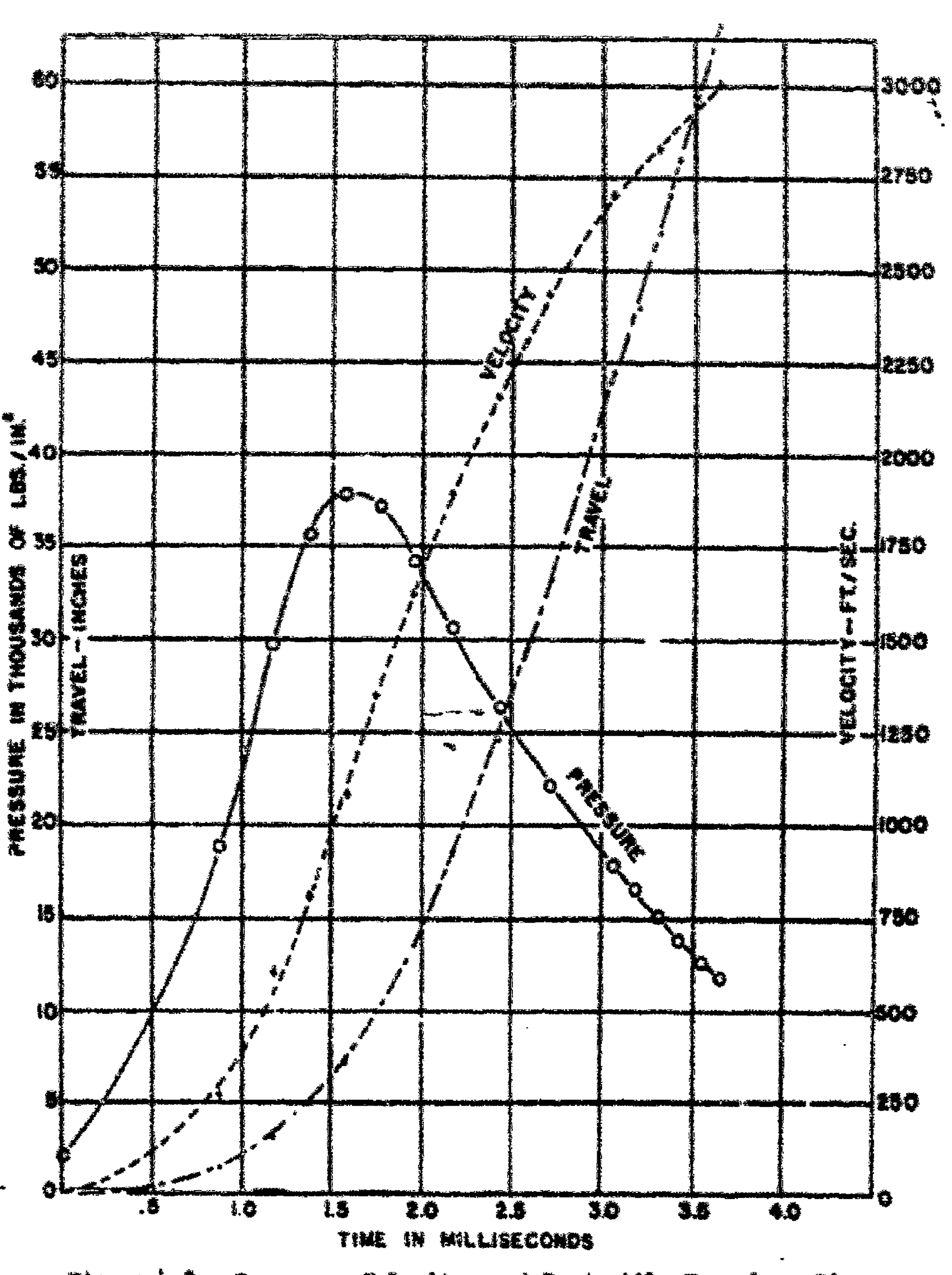

Figure 4-2. Pressure, Velocity, and Projectile Travel vs Time

5

EXTERIOR BALLISTICS

SECTION 5
EXTERIOR BALLISTICS

Projectile Shape

As outlined in Section 3, the projectile is to have two pre-engraved rotating bands. Extensive study of projectiles of this type demonstrated the advantage of the twin-band design. The addition of the forward band increases the drag by a maximum of only 1.7% (Reference 5a). However, internal yaw in the gun bore is reduced considerably, resulting in greatly improved launching conditions and decreased external yaw (Reference 5b).

In considering the type of projectile to be used (mine type with a 10 caliber radius ogive) the Type 7 appeared to be closer to the desired shape than the Type 8. The latter is a shorter version of Type 7, with the same ogive but a different base. It is understood that the 30mm British Aden projectile is considered to be of Type 8 with a form factor $i_8 = 1.218$ at a velocity of 2000 feet per second. Since the 37mm projectile has a 10 caliber radius ogive, while the Aden projectile has an ogival radius of 6 calibers, it is reasonable to assign a form factor i_7 of 1.0 to the former (Reference 5c).

Exterior Ballistic Functions

Exterior ballistic functions are to be considered at three different altitudes: sea level, 20,000 feet, and 60,000 feet.

The speed of the attacking plane V_f is as follows:

At sea level	V_f mach 1.0 = 1120 feet per second
At 20,000 feet	V_f mach 1.35 = 1400 feet per second
At 60,000 feet	V_f mach 1.50 = 1460 feet per second

The above is based on relative velocity of sound (a) being .928 at 20,000 feet and .870 at 60,000 compared to 1.0 at sea level.

The relative air density ρ is .533 at 20,000 feet, and .10 at 60,000 feet. (See Reference 5d.)

Since

$$C_7 = \frac{W}{\rho\, i_7\, d^2}$$

W = weight of projectile = 1.35 pounds

$d^2 = 1.457^2 = 2.12$ square inches

$i_7 = 1.0$

hence

$$C_7 = \frac{1.35}{1.0 \times 1.0 \times 2.12} = .635 \text{ at sea level}$$

$$C_7 = \frac{1.35}{.533 \times 1.0 \times 2.12} = 1.19 \text{ at 20,000 feet}$$

$$C_7 = \frac{1.35}{.10 \times 1.0 \times 2.12} \cong 6.35 \text{ at 60,000 feet}$$

Thus it is possible to construct ballistic tables, obtaining air travel (P) and time of flight (t) for all three altitude conditions using the air velocity (U) as one of the arguments.

Since the muzzle velocity V_o = 3000 feet per second,

U_o = 3000 + 1460 = 4460 feet per second at 60,000 feet altitude

U_o = 3000 + 1400 = 4400 feet per second at 20,000 feet altitude

U_o = 3000 + 1120 = 4120 feet per second at sea level.

Hence, total air travel $P = [S(U) - S(U_o)] \times C_7$ feet for any velocity U less than the original air velocity U_o.

The time of flight (t) is obtained in a similar manner

$$t = \left[T(U) - T(U_o)\right] \times C_7$$

The Siacci Space and Time Functions for Type 7 projectiles are used as noted in Reference 5e.

NOTE: P_1 = air travel at 60,000 feet altitude

P_2 = air travel at 20,000 feet altitude

P_3 = air travel at sea level.

t_1 = time of flight at 60,000 feet altitude

t_2 = time of flight at 20,000 feet altitude

t_3 = time of flight at sea level.

TABLE 5-1

TABLE OF SPACE AND TIME FUNCTIONS FOR 37mm PROJECTILE

U (Ft/Sec)	$S_{(U)}$ (Feet)	P_1 (Feet)	P_2 (Feet)	P_3 (Feet)	$T_{(U)}$ (Sec)	t_1 (Sec)	t_2 (Sec)	t_3 (Sec)
4460	13,647	0			2.430	0		
4450	13,703	355			2.443	.083		
4440	13,769	710			2.455	.159		
4430	13,816	1,070			2.468	.242		
4420	13,873	1,435			2.481	.325		
4410	13,920	1,740			2.493	.400		
4400	13,986	2,150	0		2.506	.483	0	
4380	14,099	2,875	135		2.532	.649	.031	
4360	14,212	3,600	269		2.558	.812	.062	
4340	14,325	4,300	404		2.584	.977	.093	
4320	14,438	5,020	539		2.610	1.145	.124	
4300	14,552	5,750	675		2.636	1.310	.154	
4280	14,665	6,450	808		2.662	1.475	.186	
4260	14,779	7,190	943		2.689	1.645	.218	
4240	14,893	7,910	1,078		2.716	1.820	.250	
4220	15,006	8,650	1,213		2.743	1.990	.282	
4200	15,120	9,350	1,350		2.770	2.165	.314	
4180	15,235	10,050	1,485		2.798	2.335	.347	
4160	15,349	10,800	1,620		2.825	2.510	.379	
4140	15,463	11,500	1,750		2.853	2.690	.413	
4120	15,577	12,240	1,890	0	2.880	2.850	.445	0
4100	15,692	13,000	2,030		2.908	3.040	.478	
4050	15,979		2,370	255	2.973		.556	.059
4000	16,266		2,710	437	3.050		.648	.108
3950	16,554		3,055	620	3.121		.730	.153
3900	16,843		3,400	805	3.196		.820	.200
3850	17,133		3,740	989	3.271		.910	.248

Sheet 1 of 3

TABLE 5-1 (CONT'D)

U (Ft/Sec)	$S_{(U)}$ (Feet)	P_1 (Feet)	P_2 (Feet)	P_3 (Feet)	$T_{(U)}$ (Sec)	t_1 (Sec)	t_2 (Sec)	t_3 (Sec)
3800	17,423		4,075	1,175	3.346		1.00	.296
3750	17,714		4,430	1,360	3.421		1.09	.344
3700	18,006		4,790	1,545	3.501		1.185	.395
3650	18,299		5,130	1,730	3.581		1.275	.445
3600	18,592		5,480	1,920	3.661		1.37	.496
3550	18,887		5,840	2,105	3.743		1.47	.548
3500	19,182		6,190	2,290	3.828		1.57	.601
3450	19,479		6,540	2,480	3.913		1.675	.654
3400	19,777		6,880	2,670	4.000		1.778	.710
3350	20,076		7,230	2,860	4.089		1.885	.767
3300	20,377		7,590	3,050	4.179		1.990	.823
3250	20,679		7,940	3,250	4.271		2.100	.884
3200	20,983		8,300	3,435	4.366		2.215	.945
3150	21,289		8,690	3,640	4.461		2.325	1.007
3100	21,596		9,050	3,820	4.560		2.44	1.070
3050	21,906		9,400	4,020	4.660		2.56	1.13
3000	22,217		9,790	4,220	4.764		2.69	1.20
2950	22,532		10,150	4,415	4.869		2.81	1.265
2900	22,848		10,550	4,615	4.979		2.94	1.33
2850	23,167		10,900	4,820	5.089		3.07	1.405
2800	23,489		11,300	5,020	5.204		3.20	1.48
2750	23,813		11,700	5,230	5.321		3.35	1.55
2700	24,141		12,050	5,440	5.441		3.48	1.63
2650	24,471		12,450	5,650	5.565		3.64	1.70
2600	24,606		12,900	5,870	5.692		3.79	1.79
2550	25,343		13,300	6,070	5.823		3.95	1.87
2500	25,484		13,700	6,300	5.958		4.12	1.955
2450	25,829		14,100	6,500	6.098		4.28	2.040

Sheet 2 of 3

TABLE 5-1 (CONT'D)

U (Ft/Sec)	$S_{(U)}$ (Feet)	P_1 (Feet)	P_2 (Feet)	P_3 (Feet)	$T_{(U)}$ (Sec)	t_1 (Sec)	t_2 (Sec)	t_3 (Sec)
2400	26,170		14,500	6,730	6.242		4.45	2.14
2350	26,531		14,900	6,950	6.391		4.61	2.23
2300	26,889		15,350	7,180	6.544		4.80	2.33
2250	27,251		15,800	7,410	6.703		4.99	2.43
2200	27,618		16,250	7,630	6.868		5.20	2.53
2150	27,990			7,900	7.040			2.65
2100	28,368			8,100	7.217			2.75
2050	28,750			8,440	7.401			2.87
2000	29,137			8,600	7.592			3.00
1950	29,530			8,850	7.790			3.12
1900	29,927			9,100	7.997			3.26
1850	30,329			9,350	8.212			3.40
1800	30,737			9,600	8.435			3.53
1750	31,149			9,870	8.667			3.67
1700	31,567			10,150	8.909			3.82
1650	31,990			10,450	9.162			3.98
1600	32,416			10,700	9.426			4.16
1550	32,852			11,000	9.700			4.34
1500	33,291			11,250	9.989			4.52

Sheet 3 of 3

Exterior Ballistics Applied to Combat Conditions

For the combat conditions outlined in Section 6, the exterior ballistic characteristics are considered at 20,000 feet altitude.

From BRL Note 807, Formula (4) page 5:

$$r_f = P - V_f t$$

The speed of the fighter (V_f) is 1400 feet per second at 20,000 feet altitude. Hence, in order to obtain P and t, use figure 5-1. These values are selected from the 20,000-foot curve for the values of future range (r_f) considered. In order to obtain U or the projectile air velocity at impact, use figure 5-2 with the value of t found from figure 5-1.

For combat conditions No. 1 and No. 2 of Section 6, the present range r is obtained according to BRL Note 807, Formula (4) page 5.

$$r = P - V_t t$$

The velocity of the target V_t is 811 feet per second for the bomber, and 1400 feet per second for the fighter. Hence for the latter case, $r = r_f$.

Formula (5), page 5 of BRL Note 807 gives the striking velocity V_s against the target on a pursuit course:

$$V_s = U - V_t$$

Hence, the table below may be used for determining the ballistic parameters for combat conditions No. 1 and No. 2 on the pursuit course attack.

TABLE 5-2

BALLISTIC PARAMETERS FOR PURSUIT ATTACK

r_f (Feet)	t (Seconds)	Bomber Target r (Feet)	P (Feet)	U (Ft/Sec)	Bomber Target V_s (Ft/Sec)	Fighter Target V_s (Ft/Sec)
1500	.535	1815	2250	4069	3258	2669
3000	1.13	3665	4580	3733	2922	2333
4500	1.82	5572	7050	3381	2570	1981
6000	2.69	7584	9760	3005	2194	1605

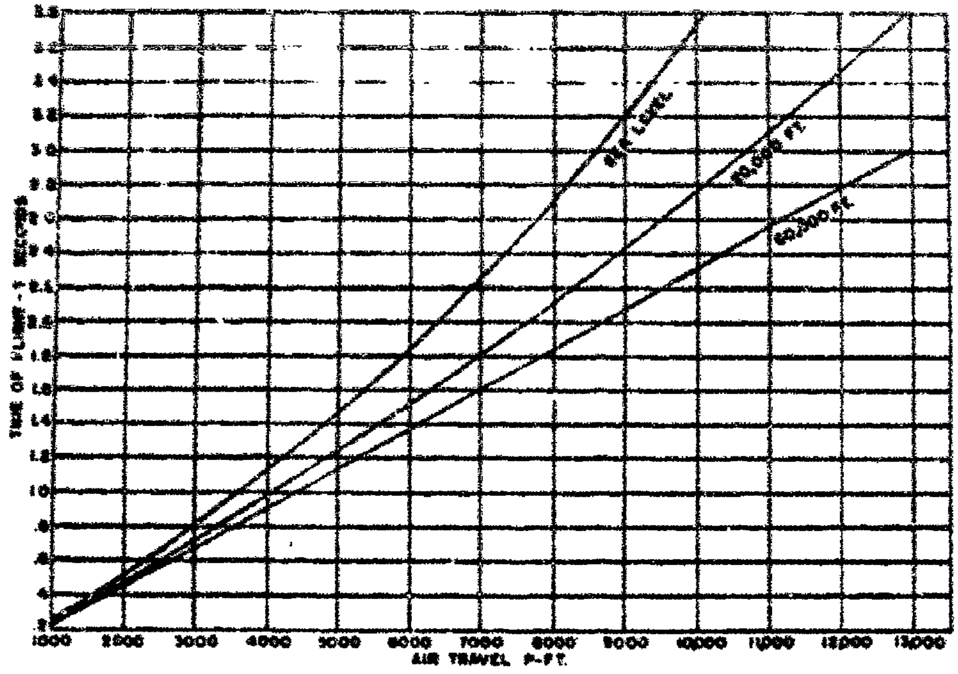

Figure 5-1. Air Travel vs Time of Flight

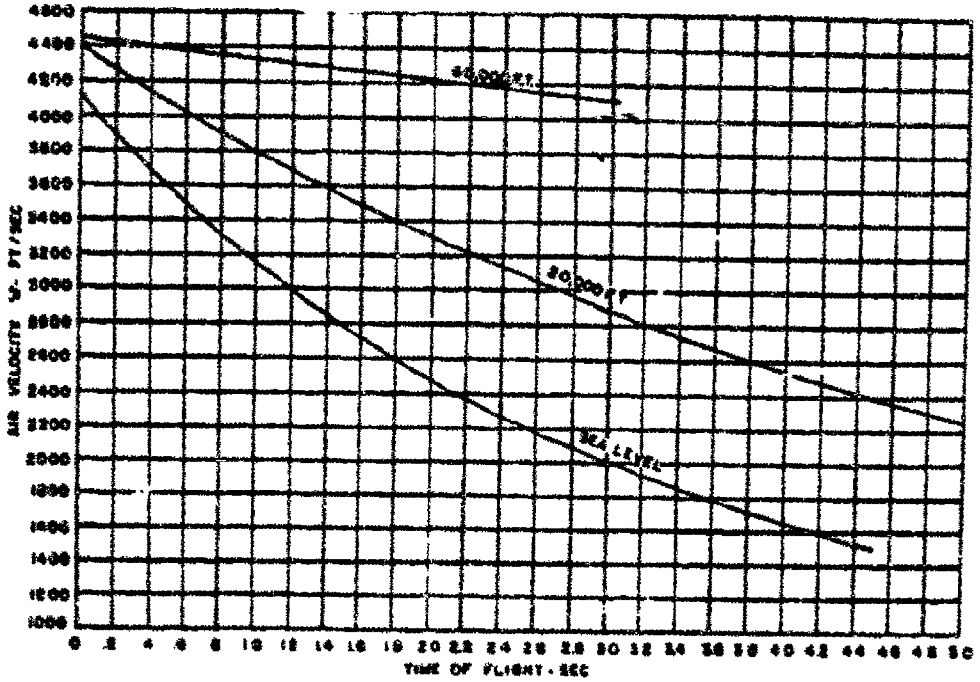

Figure 5-2. Air Velocity vs Time of Flight

Gyroscopic Stability

The gyroscopic stability was determined by the method outlined in BRL Technical Note 771, Appendix IV -- Stability Calculations.

$$S = \frac{2.27 \times 10^5 I^2}{N^2 T d^5 K_n (h - g)} \frac{V^2}{U^2}$$

This was determined at sea level conditions at the muzzle. All other conditions would show an increased stability factor, S, hence:

V = 3000 feet/second and U = 4120 feet/second

I = Axial moment of inertia, $lb\text{-}in^2$

N = Calibers per turn rifling

T = Transverse moment of inertia, $lb\text{-}in^2$

d = Caliber, inches

K_n = Normal force coefficient, non-dimensional

h = Distance from base of shell to center of pressure, calibers

g = Distance from base of shell to center of gravity, calibers

U = Initial velocity of projectile relative to air, feet per second

V = Muzzle velocity of projectile, feet per second

$$K_n = .653 + .0223a - .6139b - .0024c + .2635d + .6476 \left(\frac{1}{e}\right)$$

$$h = .0747 - .0443a + 1.019b + .8032c + .2459d + .8083 \left(\frac{1}{e}\right)$$

a = angle of boat-tail, degrees

b = length of boat-tail, calibers

c = length of cylindrical portion of body, calibers

d = length of ogive, calibers

e = ogival radius, calibers

From Section 3, the following parameters have been established.

$I = .3257 \text{ lb-in}^2$

$T = 4.7211 \text{ lb-in}^2$

$N = 25$ calibers per turn rifling

$d = 1.457$ inches

$g = 2.26$ calibers

$a = 10^{\circ}$

$b = .40$ caliber

$c = 2.36$ calibers

$d = 2.06$ calibers

$e = 10.0$ calibers

In considering the hemispherical shape of the projectile base, an equivalent boat-tailed section of 10° was selected, .40 caliber long for a and b, in order to use the outlined methods to obtain K_n and h. This is believed to be a conservative assumption.

$$K_n = .653 + .0223(10) - .6139(.40) - .0024(2.36) + .2635(2.06) + .6476\left(\frac{1}{10}\right)$$

$$K_n = 1.2321$$

$$h = .0747 - .0443(10) + 1.019(.40) + .8032(2.36) + .2459(2.06) + .8083\left(\frac{1}{10}\right)$$

$$h = 2.5275$$

$$S = \frac{2.27 \times 10^5 \times .3257^2}{25^2 \times 4.721 \times 1.457^5 \times 1.232(2.5275 - 2.26)} \times \frac{3000^2}{4120^2}$$

$$S = 2.02$$

This stability factor is satisfactory as it exceeds the accepted figure of 1.4.

References -- Section 5

From Technical Report of Div. 1, NDRC, Volume 1 -- Hypervelocity Guns:

Reference (5a) -- Paragraph 27.3.3, page 528

Reference (5b) -- Paragraph 27.3.2, page 525, and Figure 8, page 529

From Project Chore -- Application of Exterior Gun Ballistics to Aerial Combat, by J. W. Duroux. Published by University of Chicago, Ordnance Research No. 1 -- ATI-93416:

Reference (5c) -- Appendix, Standard Projectile Types

Reference (5d) -- Appendix, Table of Altitude Functions

Reference (5e) -- Table of Siacci Space and Time Functions for Type 7 Projectile

6

KILL PROBABILITIES

SECTION 6

CALCULATIONS FOR KILL PROBABILITIES OF 37 MM OPEN CHAMBER GUN

COMBAT CONDITION NO. 1

Tail Cone Lead, Pursuit Course — Fighter vs Bomber

In this case, it is assumed that the fighter is on a modified pursuit course, 10° off the tail. It is also assumed that the two planes containing the line of flight of the bomber and the fighter form an angle of 30°. The fighter is assumed to be flying a straight line rather than a curved course.

All parameters used in the P_K calculations are those set up by the combat conditions specified except those which are determined by the weapon and ammunition chosen. Thus, for example, future ranges, plane velocities, dispersion, time of firing, etc. are the same as outlined in the original rules.

Since the weapon chosen does not lend itself to firing one-third the complement of ammunition, kill probabilities have been shown for one-half and one-quarter of the total number of rounds, as well as for the full complement.

It will also be shown that a higher kill probability may be obtained by a longer time of burst for the same number of rounds, and a slightly reduced dispersion pattern. However, this is not considered in the evaluation, which is based on 1.0 second firing time and a dispersion of 5.0 mils 50% circle.

Conditions

θ = angle of pursuit off bomber nose = 170°

α = angle of intersection of planes of flight = 30°

V_f = velocity of fighter = 1400 ft/sec

V_t = velocity of target (bomber) = 811 ft/sec

V_o = muzzle velocity of gun = 3000 ft/sec

U_o = initial velocity of projectile relative to air = 4400 ft/sec

U = air velocity of projectile at impact -- ft/sec

V_s = striking velocity of projectile at impact -- ft/sec

t = time of flight to target -- seconds

r = present range -- feet

r_f = future range -- feet

P = total air travel of projectile to target -- feet

W = weight of projectile = 1.35 lb

H.E. content = 0.40 lb of MCX

φ = conversion factor relative to TNT = 1.5

σ_1^2 = 18.046 mils², σ_1 = .8496X radius of 50% circle

σ^2 = standard deviation random error (round to round) -- mils²

σ_h^2 = standard deviation systematic error (bias) -- mils²

T = time of burst = 1 second

A_v = vulnerable area -- square feet

$\left.\begin{array}{l} A \\ \frac{1}{B} \end{array}\right\}$ parameters to be used in determining H_o

H_c = probability of no hit

P_k = kill probability = $1 - H_o$

Altitude = 20,000 feet

TABLE 6-1

Summary Combat Condition No. 1

Tail Cone Lead, Pursuit Course -- Fighter vs Bomber

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
r_f (ft)	t (sec)	r (ft)	P (ft)	U (ft/sec)	V_s (ft/sec)	TNT eq (lb)	A_v (ft^2)	σ^2 ($mils^2$)	σ_h^2 ($mils^2$)
1500	.535	1815	2250	4069	3258	.779	134.0	123.00	114.43
3000	1.13	3665	4580	3733	2922	.745	128.5	509.63	432.69
4500	1.82	5572	7050	3381	2570	.712	122.5	1207.57	993.61
6000	2.69	7584	9760	3005	2194	.682	117.5	2314.[illegible]	[illegible]

	(10)	(11)	(12)	(13)	(14)	(15)	(16)
r_f (ft)	$\frac{1}{B}$	A($\frac{\text{Full}}{\text{Load}}$)	A($\frac{1/2}{\text{Load}}$)	A($\frac{1/4}{\text{Load}}$)	P_K (Full)	P_K (1/2)	P_K (1/4)
1500	1.262	53.204	26.602	13.301	.992	.978	.950
3000	1.225	13.892	6.946	3.473	.948	.885	.765
4500	1.235	5.721	2.860	1.430	.86	.72	.52
6000	1.202	2.886	1.443	.721	.72	.52	.32

For solution of columns 1--5, see page 5-7

For solution of columns 6 and 7, see page 6-5

For solution of column 8, see page 6-6

For solution of column 9, see page 6-7

For solution of column 10, see page 6-8

For solution of columns 11--13, see page 6-8

For solution of columns 14--16, see page 6-10.

For P_K curves against present and future ranges, see pages 6-48 and 6-49 respectively.

Solution of Column (6)-- TNT equivalent

From BRL Report 807, formula (11), page 6:

$$\text{lb TNT equiv} = .40 \times 1.5\left(1 + \frac{V_s^2}{8 \times 10^7 \times 1.5 \times \frac{.4}{1.35}}\right) = .60\left(1 + \frac{V_s^2}{3.55 \times 10^7}\right)$$

For $r_f = 1500$

$V_s = 3258$ ft/sec TNT equiv $= .60\left(1 + \frac{3258^2}{3.55 \times 10^7}\right)$

$= .60 \times 1.299 = .779$ lb.

$r_f = 3000$

$V_s = 2922$ ft/sec TNT equiv $= .60\left(1 + \frac{2922^2}{3.55 \times 10^7}\right)$

$= .60 \times 1.241 = .745$ lb.

$r_f = 4500$

$V_s = 2570$ ft/sec TNT equiv $= .60\left(1 + \frac{2570^2}{3.55 \times 10^7}\right)$

$= .60 \times 1.186 = .712$ lb.

$r_f = 6000$

$V_s = 2194$ ft/sec TNT equiv $= .60\left(1 + \frac{2194^2}{3.55 \times 10^7}\right)$

$= .60 \times 1.136 = .682$ lb.

Solution of Column (7)

From BRL Report 807, Curve Figure 1: Vulnerable area of bomber target A_v is as follows:

TNT -- lb.	A_v -- Square Feet
.779	134.0
.745	128.5
.712	112.5
.682	117.5

Solution of Column (8) -- σ^2 (as corrected under date of 8-3-53)

From BRL Report 807, formula on page 7:

$$\sigma^2 = \left[\sigma_1^2 + \left(\frac{T}{3 + T}\right) 25 \right] \left(\frac{P}{1000}\right)^2$$

$\sigma_1^2 = 18.046$, $T = 1$ second

$$\sigma^2 = 24.296 \left(\frac{P}{1000}\right)^2$$

For $r_f = 1500$ feet, $P = 2250$ feet

$$\sigma^2 = 24.296 \times 2.25^2 = 123.00 \text{ mils}^2$$

For $r_f = 3000$ feet, $P = 4580$ feet

$$\sigma^2 = 24.296 \times 4.58^2 = 509.63 \text{ mils}^2$$

For $r_f = 4500$ feet, $P = 7050$ feet

$$\sigma^2 = 24.296 \times 7.05^2 = 1207.57 \text{ mils}^2$$

For $r_f = 6000$ feet, $P = 9760$ feet

$$\sigma^2 = 24.296 \times 9.76^2 = 2314.39 \text{ mils}^2$$

<u>Solution of Column (9) — σ_h^2</u> (as corrected under date of 8-3-53)

From BRL Report 807, formula on page 7:

$$\sigma_h^2 = 8100 \left(\frac{V_t \times t \times \sin\theta}{r}\right)^2 \left[1 + \left(\frac{r}{3000}\right)^2\right] + \left(\frac{3}{3+T}\right) 25 \left(\frac{P}{1000}\right)^2$$

$\sin 10^\circ = .1736$, $V_t = 811$ ft/sec $T = 1$ sec.

For $r_f = 1500$ feet, $t = .535$, $r = 1815$ feet, $P = 2250$ feet

$$\sigma_h^2 = 8100 \left(\frac{811 \times .535 \times .1736}{1815}\right)^2 \left[1 + \left(\frac{1815}{3000}\right)^2\right] + 18.8 \left(\frac{2250}{1000}\right)^2$$

$= 8100 \times .00173 \times 1.366 + 95.18$

$= 19.25 + 95.18 = 114.43$ mils^2

For $r_f = 3000$ feet, $t = 1.13$, $r = 3665$ feet, P ; 4580 feet

$$\sigma_h^2 = 8100 \left(\frac{811 \times 1.13 \times .1736}{3665}\right)^2 \left[1 + \left(\frac{3665}{3000}\right)^2\right] + 18.8 \left(\frac{4580}{1000}\right)^2$$

$= 8100 \times .00189 \times 2.493 + 394.35$

$= 38.25 + 394.35 = 432.60$ mils^2

For $r_f = 4500$ feet, $t = 1.82$, $r = 5572$ feet, $P = 7050$ feet

$$\sigma_h^2 = 8100 \left(\frac{811 \times 1.82 \times .1736}{5572}\right)^2 \left[1 + \left(\frac{5572}{3000}\right)^2\right] + 18.8 \left(\frac{7050}{1000}\right)^2$$

$= 8100 \times .00212 \times 3.44 + 934.4.$

$= 59.20 + 934.41 = 993.61$ mils^2

For $r_f = 6000$ feet, $t = 2.69$, $r = 7584$ feet, $P = 9760$ feet

$$\sigma_h^2 = 8100 \left(\frac{811 \times 2.69 \times .1736}{7584}\right)^2 \left[1 + \left(\frac{7584}{3000}\right)^2\right] + 18.8 \left(\frac{9760}{1000}\right)^2$$

$= 8100 \times .0025 \times 7.391 + 1790.85$

$= 150.0 + 1790.85 = 1940.85$ mils^2

Solution of Column (10) -- $\frac{1}{B}$

From BRL Report 807, formula on page 7:

$$\frac{1}{B} = \frac{A_v + 2\pi\sigma^2}{2\pi\sigma_h^2}$$

For r_f = 1500 feet, $A_v = 134.0$, $\sigma^2 = 123.0$, $\sigma_h^2 = 114.43$

$$\frac{1}{B} = \frac{134.0 + 6.282 \times 123.0}{6.28 \times 114.43} = 1.262$$

For r_f = 3000, $A_v = 128.5$, $\sigma^2 = 509.63$, $\sigma_h^2 = 432.60$

$$\frac{1}{B} = \frac{128.5 + 6.282 \times 509.63}{6.282 \times 432.60} = 1.225$$

For r_f = 4500 feet, $A_v = 122.5$, $\sigma^2 = 1207.57$, $\sigma_h^2 = 993.61$

$$\frac{1}{B} = \frac{122.5 + 6.282 \times 1207.57}{6.282 \times 993.61} = 1.235$$

For r_f = 6000 feet, $A_v = 117.5$, $\sigma^2 = 2314.39$, $\sigma_h^2 = 1940.85$

$$\frac{1}{B} = \frac{117.5 + 6.28 \times 2314.39}{6.282 \times 1940.85} = 1.202$$

Solution of Columns (11)--(13)

From BRL Report 807, formula on page 7:

$$A = \frac{nA_v}{A_v + 2\pi\sigma^2}$$

n = 360 (Full load); 180 (1/2 load); 90 (1/4 load)

For r_f = 1500 feet, $A_v = 134.0$, $\sigma^2 = 123.0$

$$\underset{(n = 360)}{A} = \frac{360 \times 134.0}{134.0 + 6.282 \times 123.0} = 53.204$$

$$\underset{(n = 180)}{A} = \frac{53.204}{2} = 26.602$$

$$\underset{(n = 90)}{A} = \frac{53.204}{4} = 13.301$$

For r_f = 3000 feet, $A_v = 128.5$, $\sigma^2 = 509.63$

$$\underset{(n = 360)}{A} = \frac{360 \times 128.5}{128.5 + 6.282 \times 509.63} = 13.892$$

$$\underset{(n = 180)}{A} = \frac{13.892}{2} = 6.946$$

$$\underset{(n = 90)}{A} = \frac{13.892}{4} = 3.473$$

For r_f = 4500 feet, $A_v = 122.5$, $\sigma^2 = 1207.57$

$$\underset{(n = 360)}{A} = \frac{360 \times 122.5}{122.5 + 6.282 \times 1207.57} = 5.721$$

$$\underset{(n = 180)}{A} = \frac{5.721}{2} = 2.860$$

$$\underset{(n = 90)}{A} = \frac{5.721}{4} = 1.430$$

For r_f = 6000 feet, $A_v = 117.5$, $\sigma^2 = 2314.39$

$$\underset{(n = 360)}{A} = \frac{360 \times 117.5}{117.5 + 6.282 \times 2314.39} = 2.886$$

$$\underset{(n = 180)}{A} = \frac{2.886}{2} = 1.443$$

$$\underset{(n = 90)}{A} = \frac{2.886}{4} = .721$$

Solution to Columns (14)--(16)

Having the parameters A and 1/B, it becomes a simple matter to obtain P_K by finding H_o from Figure 47 of BRL Report MR462. Since H_o is the probability of no hits on a vulnerable area, P_K becomes $1 - H_o$.

For	Full load,	$\frac{1}{B} = 1.262$,	A = 53.204,	H_o = .008
r_f = 1500	Half load,	$\frac{1}{B} = 1.262$,	A = 26.602,	H_o = .022
	Quarter load,	$\frac{1}{B} = 1.262$,	A = 13.301,	H_o = .050
For	Full load,	$\frac{1}{B} = 1.225$,	A = 13.892,	H_o = .052
r_f = 3000	Half load,	$\frac{1}{B} = 1.225$,	A = 6.946,	H_o = .115
	Quarter load,	$\frac{1}{B} = 1.225$,	A = 3.473,	H_o = .235
For	Full load,	$\frac{1}{B} = 1.235$,	A = 5.721,	H_o = .14
r_f = 4500	Half load,	$\frac{1}{B} = 1.235$,	A = 2.860,	H_o = .28
	Quarter load,	$\frac{1}{B} = 1.235$,	A = 1.430,	H_o = .48
For	Full load,	$\frac{1}{B} = 1.202$,	A = 2.886,	H_o = .28
r_f = 6000	Half load,	$\frac{1}{B} = 1.202$,	A = 1.443,	H_o = .48
	Quarter load,	$\frac{1}{B} = 1.202$,	A = .721,	H_o = .68

The previous combat conditions reveal high kill probabilities for future ranges up to and including 4500 feet.

Further examination reveals that the kill probability may be improved at r_f = 6000 feet by decreasing the rate of fire, and decreasing the radius of the 50% circle from 5.0 mils to some lower figure. That is, the same number of rounds fired over a longer period of time will improve P_K considerably.

The following combinations will be considered for r_f = 6000 feet with all other conditions remaining the same as the original.

a. T = 2.0 sec, radius 50% circle = 5.0 mils

b. T = 2.0 sec, radius 50% circle = 4.0 mils

c. T = 2.0 sec, radius 50% circle = 3.0 mils

d. T = 3.0 sec, radius 50% circle = 5.0 mils

e. T = 3.0 sec, radius 50% circle = 4.0 mils

f. T = 3.0 sec, radius 50% circle = 3.0 mils

<u>Condition a.</u> T = 2.0 seconds, CEP = 5 mils, $\sigma_i^2 = 18.046$

(See page 6-6) $\sigma^2 = (18.046 + \frac{2}{5} \times 25) \times 9.76^2 = 2671.61$

(See page 6-7) $\sigma_h^2 = 150.0 + (\frac{3}{5} \times 25) \times 9.76^2 = 1578.87$

(See page 6-8) $\frac{1}{B} = \frac{117.5 + 6.282 \times 2671.61}{6.282 \times 1578.87} = 1.704$

(See page 6-9)

		H_0	P_K
$\underset{(n = 360)}{A}$	$= \frac{360 \times 117.5}{117.5 + 6.282 \times 2671.61} = 2.503$	.235	.765
$\underset{(n = 180)}{A}$	$= \frac{2.503}{2} = 1.252$	.45	.55
$\underset{(n = 90)}{A}$	$= \frac{2.503}{4} = .626$	.68	.32

Condition b. $T = 2.0$ seconds, CEP $= 4.0$ mils, $\sigma_1^2 = 11.549$

$$\sigma^2 = (11.549 + \frac{2}{5} \times 25) \times 9.76^2 = 2052.71$$

See Condition a.

$$\sigma_h^2 = 1578.87$$

$$\frac{1}{B} = \frac{117.5 + 6.282 \times 2052.71}{6.282 \times 1578.87} = 1.312$$

	H_o	P_K
$\underset{(n = 360)}{A} = \frac{360 \times 117.5}{117.5 + 6.282 \times 2052.71} = 3.250$	.235	.765
$\underset{(n = 180)}{A} = \frac{3.250}{2} = 1.625$	.425	.575
$\underset{(n = 90)}{A} = \frac{3.250}{4} = .812$	.64	.36

- -

Condition c. $T = 2.0$ seconds, CEP $= 3.0$ mils, $\sigma_1^2 = 6.497$

$$\sigma^2 = (6.497 + \frac{2}{5} \times 25) \times 9.76^2 = 1571.47$$

See Condition a.

$$\sigma_h^2 = 1578.87$$

$$\frac{1}{B} = \frac{117.5 + 6.282 \times 1571.47}{6.282 \times 1578.87} = 1.007$$

	H_o	P_K
$\underset{(n = 360)}{A} = \frac{360 \times 117.5}{117.5 + 6.282 \times 1571.47} = 4.234$	.23	.77
$\underset{(n = 180)}{A} = \frac{4.234}{2} = 2.117$	.405	.595
$\underset{(n = 90)}{A} = \frac{4.234}{4} = 1.058$	.605	.395

- -

Condition d. $T = 3.0$ seconds, CEP = 5.0 mils, $\sigma_1^2 = 18.046$

$$\sigma^2 = (18.046 + \frac{3}{6} \times 25) \times 9.76^2 = 2909.75$$

See page 6-7.

$$\sigma_h^2 = 150.0 + (\frac{3}{6} \times 25) \times 9.76^2 = 1340.72$$

$$\frac{1}{B} = \frac{117.5 + 6.282 \times 2909.75}{6.282 \times 1340.72} = 2.184$$

	H_o	P_K
$\underset{(n = 360)}{A} = \dfrac{360 \times 117.5}{117.5 + 6.282 \times 2909.75} = 2.30$	.21	.79
$\underset{(n = 180)}{A} = \dfrac{2.30}{2} = 1.15$	.43	.57
$\underset{(n = 90)}{A} = \dfrac{2.30}{4} = .575$	.67	.33

Condition e. $T = 3.0$ seconds, CEP = 4.0 mils $\sigma_1^2 = 11.549$

$$\sigma^2 = (11.549 + \frac{3}{6} \times 25) \times 9.76^2 = 2290.86$$

See Condition d.

$$\sigma_h^2 = 1340.72$$

$$\frac{1}{B} = \frac{117.5 + 6.282 \times 2290.86}{6.282 \times 1340.72} = 1.722$$

	H_o	P_K
$\underset{(n = 360)}{A} = \dfrac{360 \times 117.5}{117.5 + 6.282 \times 2290.86} = 2.915$	.195	.805
$\underset{(n = 180)}{A} = \dfrac{2.915}{2} = 1.457$	.40	.60
$\underset{(n = 90)}{A} = \dfrac{2.915}{4} = .729$	.64	.36

Condition f. $T = 3.0$ seconds, $CEP = 3.0$ mils, $\sigma_1^2 = 6.497$

$$\sigma^2 = (6.497 + \frac{3}{8} \times 25) \times 9.76^2 = 1809.62$$

See Condition d.

$$\sigma_h^2 = 1340.72$$

$$\frac{1}{B} = \frac{117.5 + 6.282 \times 1809.62}{6.282 \times 1340.72} = 1.363$$

	H_o	P_K
$\underset{(n = 360)}{A} = \frac{360 \times 117.5}{117.5 + 6.282 \times 1809.62} = 3.683$	.195	.805
$\underset{(n = 180)}{A} = \frac{3.683}{2} = 1.842$	.39	.61
$\underset{(n = 90)}{A} = \frac{3.683}{4} = .921$	.60	.40

COMBAT CONDITION NO. 2

Tail Cone Lead -- Pursuit Course

Fighter vs Fighter

All conditions are the same as for Condition No. 1 (Bomber Target) except that the speed of the fighter target V_t is the same as own speed of 1400 feet per second.

Hence $r_f = r$.

See p. 6-2 for explanation of symbols and values.

TABLE 6-2

Summary Combat Condition No. 2

Tail Cone Lead, Pursuit Course -- Fighter vs Fighter

r (ft)	(1) t (sec)	(2) P (ft)	(3) U (ft/sec)	(4) V_s (ft/sec)	(5) TNT Eq (lb)	(6) A_v (ft^2)	(7) σ^2	(8) σ_h^2
1500	.535	2250	4069	2669	.721	76.0	123.00	171.22
3000	1.13	4580	3733	2333	.692	75.5	509.63	529.78
4500	1.82	7050	3381	1981	.667	75.3	1207.57	1188.71
6000	2.69	9760	3005	1605	.644	75.0	2314.39	2272.80

r (ft)	(9) $\frac{1}{B}$	(10) A (Full Load)	(11) A (1/2 Load)	(12) A (1/4 Load)	(13) P_K (Full)	(14) P_K (1/2)	(15) P_K (1/4)
1500	.789	32.238	16.119	8.060	.94	.90	.82
3000	.985	8.294	4.147	2.074	.87	.76	.59
4500	1.026	3.538	1.769	.885	.74	.55	.35
6000	1.023	1.848	.924	.462	.56	.36	.20

For solution of columns 1--4, see page 5-7

For solution of columns 5 and 6, see page 6-17

For solution of column 7, see page 6-18

For solution of column 8, see page 6-19

For solution of column 9, see page 6-20

For solution of columns 10--12, see page 6-21

For solution of columns 13--15, see page 6-22

For P_K curves see page 6-50.

Solution of Column (5) -- TNT Equivalent

From BRL Report 807, formula (11) page 6:

$$\text{lb TNT equiv} = .40 \times 1.5 \left(1 + \frac{V_s^2}{8 \times 10^7 \times 1.5 \times \frac{.4}{1.35}}\right)$$

$$= .60 \left(1 + \frac{V_s^2}{3.55 \times 10^7}\right)$$

For r = 1500 feet, V_s = 2669 ft/sec

$$\text{TNT equiv} = .60 \left(1 + \frac{2669^2}{3.55 \times 10^7}\right)$$

$$= .60 \times 1.201 = .721 \text{ lb.}$$

---

r = 3000 feet, V_s = 2333 ft/sec

$$\text{TNT equiv} = .60 \left(1 + \frac{2333^2}{3.55 \times 10^7}\right)$$

$$= .60 \times 1.154 = .692 \text{ lb.}$$

---

r = 4500 feet, V_s = 1981 ft/sec

$$\text{TNT equiv} = .60 \left(1 + \frac{1981^2}{3.55 \times 10^7}\right)$$

$$= .60 \times 1.111 = .667 \text{ lb.}$$

---

r = 6000 feet, V_s = 1605 ft/sec

$$\text{TNT equiv} = .60 \left(1 + \frac{1605^2}{3.55 \times 10^7}\right)$$

$$= .60 \times 1.073 = .644 \text{ lb.}$$

Solution of Column (6)

From BRL Report 807, Curve Figure 2:

Vulnerable area of fighter target A_v is as follows:

TNT -- lb	A_v -- Square Feet
.721	76.0
.692	75.5
.667	75.3
.644	75.0

Solution of Column (7) -- σ^2

From BRL Report 807, formula on page 7:

$$\sigma^2 = \left[\sigma_1^2 + \left(\frac{T}{3 + T}\right) 25\right] \left(\frac{P}{1000}\right)^2$$

$\sigma_1^2 = 18.046$ $\quad\quad$ $T = 1.0$ sec.

$$\sigma^2 = 24.296 \left(\frac{P}{1000}\right)^2$$

Hence σ^2 for fighter target is same as for bomber target (see page 6-6).

Solution of Column (8) -- σ_h^2

From BRL Report 807, Formula on page 7 (as corrected):

$$\sigma_h^2 = 8100 \left(\frac{V_t \times t \times \sin\theta}{r}\right)^2 \left[1 + \left(\frac{1}{3000}\right)^2\right] + \left(\frac{3}{3 + T}\right) 25 \left(\frac{P}{1000}\right)^2$$

$\sin 10^\circ = .1736$, $V_t = 1400$ ft/sec, $T = 1$ sec.

For $r = 1500$ feet, $t = .535$ sec, $P = 2250$ feet

$$\sigma_h^2 = 8100 \left(\frac{1400 \times .535 \times .1736}{1500}\right)^2 \left[1 + \left(\frac{1500}{3000}\right)^2\right] + 18.8 \left(\frac{2250}{1000}\right)^2$$

$= 8100 \times .00751 \times 1.25 + 95.18$

$= 76.04 + 95.18 = 171.22$

For $r = 3000$ feet $t = 1.13$ sec, $P = 4580$ feet

$$\sigma_h^2 = 8100 \left(\frac{1400 \times 1.13 \times .1736}{3000}\right)^2 \left[1 + \left(\frac{3000}{3000}\right)^2\right] + 18.8 \left(\frac{4580}{1000}\right)^2$$

$= 8100 \times .00836 \times 2 + 394.35$

$= 135.43 + 394.35 =$ 5[illegible].78

For $r = 4500$ feet $t = 1.82$ sec, $P = 7050$ feet

$$\sigma_h^2 = 8100 \left(\frac{1400 \times 1.82 \times .1736}{4500}\right)^2 \left[1 + \left(\frac{4500}{3000}\right)^2\right] + 18.8 \left(\frac{7050}{1000}\right)^2$$

$= 8100 \times .00966 \times 3.25 + 934.41$

$= 254.30 + 934.41 = 1188.71$

For $r = 6000$ feet $t = 2.69$ sec, $P = 9760$ feet

$$\sigma_h^2 = 8100 \left(\frac{1400 \times 2.69 \times .1736}{6000}\right)^2 \left[1 + \left(\frac{6000}{3000}\right)^2\right] + 18.8 \left(\frac{9760}{1000}\right)^2$$

$= 8100 \times .0119 \times 5 + 1790.85$

$= 481.95 + 1790.85 = 2272.80$

Solution of Column (9) -- $\frac{1}{B}$

From BRL Report 807, formula on page 7:

$$\frac{1}{B} = \frac{A_v + 2\pi\sigma^2}{2\pi\sigma_h^2}$$

- -

For $r = 1500$ ft, $A_v = 76.0$, $\sigma^2 = 123.0$, $\sigma_h^2 = 171.22$

$$\frac{1}{B} = \frac{76.0 + 6.282 \times 123.0}{6.282 \times 171.22} = .789$$

- -

For $r = 3000$ feet, $A_v = 75.5$, $\sigma^2 = 509.63$, $\sigma_h^2 = 529.78$

$$\frac{1}{B} = \frac{75.5 + 6.282 \times 509.63}{6.282 \times 529.78} = .985$$

- -

For $r = 4500$ feet, $A_v = 75.3$, $\sigma^2 = 1207.57$, $\sigma_h^2 = 1188.71$

$$\frac{1}{B} = \frac{75.3 + 6.282 \times 1207.57}{6.282 \times 1188.71} = 1.026$$

- -

For $r = 6000$ feet, $A_v = 75.0$, $\sigma^2 = 2314.39$, $\sigma_h^2 = 2272.80$

$$\frac{1}{B} = \frac{75.0 + 6.282 \times 2314.39}{6.282 \times 2272.80} = 1.023$$

- -

Solution of Columns (10)--(12)

From BRL Report 807, formula on page 7:

$$A = \frac{nA_v}{A_v + 2\pi\sigma^2}$$

n = 360 (full load); 180 (1/2 load); 90 (1/4 load)

For r = 1500 feet, $A_v = 76.0$, $\sigma^2 = 123.0$

$$A_{(n = 360)} = \frac{360 \times 76.0}{76.0 + 6.282 \times 123.0} = 32.238$$

$$A_{(n = 180)} = \frac{32.238}{2} = 16.119$$

$$A_{(n = 90)} = \frac{32.238}{4} = 8.060$$

For r = 3000 feet, $A_v = 75.5$, $\sigma^2 = 509.63$

$$A_{(n = 360)} = \frac{360 \times 75.5}{75.5 + 6.282 \times 509.63} = 8.294$$

$$A_{(n = 180)} = \frac{8.294}{2} = 4.147$$

$$A_{(n = 90)} = \frac{8.294}{4} = 2.074$$

For r = 4500 feet, $A_v = 75.3$, $\sigma^2 = 1207.57$

$$A_{(n = 360)} = \frac{360 \times 75.3}{75.3 + 6.282 \times 1207.57} = 3.538$$

$$A_{(n = 180)} = \frac{3.538}{2} = 1.769$$

$$A_{(n = 90)} = \frac{3.538}{4} = .885$$

For r = 6000 feet, $A_1 = 75.0$, $\sigma^2 = 2314.39$

$$\underset{(n = 360)}{A} = \frac{360 \times 75.0}{75.0 + 6.282 \times 2314.39} = 1.848$$

$$\underset{(n = 180)}{A} = \frac{1.848}{2} = .924$$

$$\underset{(n = 90)}{A} = \frac{1.848}{4} = .462$$

Solution of Columns (13) -- (15)

Obtain H_o from Figure 47 of BRL Report MR 462.

$P_K = 1 - H_o$

For r = 1500 feet:	Full load	$\frac{1}{B} = .789$	A = 32.238	$H_o = .06$
	Half load	$\frac{1}{B} = .789$	A = 16.119	$H_o = .10$
	Quarter load	$\frac{1}{B} = .789$	A = 8.060	$H_o = .18$
For r = 3000 feet	Full load	$\frac{1}{B} = .985$	A = 8.294	$H_o = .127$
	Half load	$\frac{1}{B} = .985$	A = 4.147	$H_o = .24$
	Quarter load	$\frac{1}{B} = .985$	A = 2.074	$H_o = .41$
For r = 4500 feet	Full load	$\frac{1}{B} = 1.026$	A = 3.538	$H_o = .26$
	Half load	$\frac{1}{B} = 1.026$	A - 1.769	$H_o = .455$
	Quarter load	$\frac{1}{B} = 1.026$	A = .885	$H_o = .65$
For r = 6000 feet	Full load	$\frac{1}{B} = 1.023$	A = 1.848	$H_o = .44$
	Half load	$\frac{1}{B} = 1.023$	A = .924	$H_o = .64$
	Quarter load	$\frac{1}{B} = 1.023$	A = .462	$H_o = .80$

As in the case of the bomber target (p. 6-11), P_K may be increased by longer time of firing and decreased radius of 50% circle.

The following combinations will be considered for r = 6000 feet.

a. T = 2.0 seconds, radius of 50% circle = 3.0 mils

b. T = 3.0 seconds, radius of 50% circle = 3.0 mils

<u>Condition a.</u> T = 2.0 sec, CEP = 3 mils, $\sigma_1^2 = 6.497$

$$\sigma^2 = (6.497 + \frac{2}{5} \times 25) \times 9.76^2 = 1571.47$$

See page 6-19.

$$\sigma_h^2 = 481.95 + (\frac{2}{5} \times 25) \times 9.76^2 = 1910.82$$

$$\frac{1}{B} = \frac{75.0 + 6.282 \times 1571.47}{6.282 \times 1910.82} = .829$$

	N_o	P_K
$\underset{(n = 360)}{A} = \frac{360 \times 75.0}{75.0 + 6.282 \times 1571.47} = 2.714$	.385	.615
$\underset{(n = 180)}{A} = \frac{2.714}{2} = 1.357$	.58	.42
$\underset{(n = 90)}{A} = \frac{2.714}{4} = .679$	.75	.25

Condition b. $T = 3.0$ sec, CEP = 3.0 mils $\sigma_1^2 = 6.497$

$$\sigma^2 = (6.497 + \frac{3}{6} \times 25) \times 9.76^2 = 1809.62$$

$$\sigma_h^2 = 481.95 + (\frac{3}{6} \times 25) \times 9.76^2 = 1672.67$$

$$\frac{1}{B} = \frac{75.0 + 6.282 \times 1809.62}{6.282 \times 1672.67} = 1.089$$

	H_o	P_K
$\underset{(n = 360)}{A} = \frac{360 \times 75.0}{75.0 + 6.282 \times 1809.62} = 2.36$	.385	.615
$\underset{(n = 180)}{A} = \frac{2.36}{2} = 1.18$	.56	.44
$\underset{(n = 90)}{A} = \frac{2.36}{4} = .59$	.75	.25

COMBAT CONDITION NO. 3

Offset Collision Course -- Fighter vs Bomber

All conditions and symbols are the same as for Pursuit Attack (page 6-2) except as follows.

α = angle between attacker's path and target path = 45°

$\omega = 0^\circ$

$\bar{t}$ = time in seconds corresponding to $U' = .98\,U_o$

α' = angle between projectile path relative to target at impact and target path -- degrees

λ = lead angle at time of fire -- degrees

σ_x^2 = standard deviation of horizontal bias -- mils^2

σ_y^2 = standard deviation in vertical direction -- mils^2

$\left.\begin{matrix}\sigma_H^2\\ \sigma_V^2\end{matrix}\right\}$ Horizontal and vertical components of round to round dispersion -- mils^2

$\left.\begin{matrix}a\\ b\end{matrix}\right\}$ Semi-axes of an ellipse equivalent to vulnerable target area -- feet

E_o = expected number of kills if there were no bias

C_1 thru C_7 = Components of horizontal bias standard deviations due to imperfect input data, residual tracking errors, and mechanism imperfections

R = ratio of H.E. filler weight to projectile weight

A_v' = vulnerable area read from Curve 1 of BRL Report 807 for $\alpha' = 45^\circ$ -- square feet

A_v = vulnerable area for any α' -- square feet.

TABLE 6-3

Summary Combat Condition No. 3

Offset Collision Course -- 45° Off Tail, Fighter vs Bomber

Line No.	r_f -- feet	1500	3000	4500	6000
1 (p. 6-4)	t -- sec	.535	1.13	1.82	2.69
2 (p. 6-4)	P -- ft	2250	4580	7050	9760
3 (p. 6-4)	U -- ft/sec	4069	3733	3381	3005
4 (p. 6-28)	V_s -- ft/sec	3542	3211	2865	2499
5 (p. 6-29)	r -- ft	1966	3987	6095	8370
6 (p. 6-30)	sin α'	.81219	.82186	.83431	.85014
7 (p. 6-30)	sin λ	.15603	.16251	.17121	.18428
8 (p. 6-31)	TNT equiv -- lb	.810	.774	.738	.708
9 (BRL 807 Fig. 1)	A_v' -- sq ft	290	282.5	275.0	267.5
10 (p. 6-31)	A_v -- sq ft	333.0	328.35	324.42	321.5
11 (p. 6-32)	C_1	-38.59	-38.80	-38.92	-38.81
12 (p. 6-33)	C_2	36.59	66.15	88.13	101.83
13 (p. 6-34)	C_3	25.89	100.72	221.78	392.17
14 (p. 6-35)	C_4	-.981	-5.16	-13.36	-27.04
15 (p. 6-36)	C_5	51.37	97.41	145.63	197.14
16 (p. 6-36)	C_6	27.42	50.46	69.08	82.75
17 (p. 6-37)	C_7	80.07	69.99	59.43	48.15
18 (p. 6-38)	ΣC_i^2	13,301	32,985	85,150	214,424

Line No.	r_f -- feet	1500	3000	4500	6000
19 (p. 6-38)	σ_x^2	348.49	1052.25	3526	11,300
20 (p. 6-38)	σ_y^2	126.56	524.41	1242.5	2381.4
21 (p. 6-39)	σ_H^2	88.97	366.42	860.97	1627.95
22 (p. 6-39)	σ_V^2	91.35	378.53	896.89	1719.06
23 (p. 6-40)	a	32.83	32.83	32.90	32.92
24 (p. 6-40)	b	3.23	3.19	3.14	3.11
25 (p. 6-41)	τ_H^2	32,424	30,807	29,019	27,010
26 (p. 6-42)	τ_V^2	193.13	767.23	1803.68	3447.74
27 (p. 6-43)	E ($\hat{h}$ = 360)	15.258	7.756	5.132	3.819
28 (p. 6-43)	E ($\hat{h}$ = 180)	7.629	3.878	2.566	1.910
29 (p. 6-43)	E ($\hat{h}$ = 90)	3.814	1.939	1.283	.955
30 (p. 6-44)	C_x	40.68	14.64	4.115	1.195
31 (p. 6-44)	C_y	.763	.732	.726	.724
32 (p. 6-45)	P_K ($\hat{h}$ = 360)	.99	.92	.86	.74
33 (p. 6-45)	P_K ($\hat{h}$ = 180)	.95	.84	.70	.57
34 (p. 6-45)	P_K ($\hat{h}$ = 90)	.91	.71	.52	.37

For P_K curves see Figure 6-4, page 6-51.

Solution of Line(4)-- V_s -- Striking Velocity

From BRL 807, Formula (6), page 5:

$$V_s = (V_t^2 + U^2 - 2UV_t \cos \alpha)^{1/2}$$

V_t = 811 ft/sec; $\cos \alpha$ = .707

---

r_f = 1500 feet, U = 4069 feet per second

$$V_s = (811^2 + 4069^2 - 2 \times 4069 \times 811 \times .707)^{1/2}$$

= 3542 feet per second

---

r_f = 3000 feet, U = 3733 feet per second

$$V_s = (811^2 + 3733^2 - 2 \times 3733 \times 811 \times .707)^{1/2}$$

= 3211 feet per second

---

r_f = 4500 feet, U = 3381 feet per second

$$V_s = (811^2 + 3381^2 - 2 \times 3381 \times 811 \times .707)^{1/2}$$

= 2865 feet per second

---

r_f = 6000 feet, U = 3005 feet per second

$$V_s = 811^2 + 3005^2 - 2 \times 3005 \times 811 \times .707)^{1/2}$$

= 2499 feet per second

Solution of Line (5) — r — Present Range

From BRL 807, formula (7) page 5:

$$r = \left[(V_t\ t)^2 + (r_f + V_f t)^2 - 2V_t\ t(r_f + V_f\ t)\ \cos\ \sigma\right]^{1/2}$$

---

r_f = 1500 feet, t = .535 second

$$r = \left[(811 \times .535)^2 + (1500 + 1400 \times .535)^2 - 2 \times 811 \times .535 \times .707\ (1500 + 1400 \times .535)\right]^{1/2}$$

= 1966 feet

---

r_f = 3000 feet t = 1.13 seconds

$$r = \left[(811 \times 1.13)^2 + (3000 + 1400 \times 1.13)^2 - 2 \times 811 \times 1.13 \times .707\ (3000 + 1400 \times 1.13)\right]^{1/2}$$

= 3987 feet

---

r_f = 4500 feet, t = 1.82 seconds

$$r = \left[(811 \times 1.82)^2 + (4500 + 1400 \times 1.82)^2 - 2 \times 811 \times 1.82 \times .707\ (4500 + 1400 \times 1.82)\right]^{1/2}$$

= 6095 feet

---

r_f = 6000 feet, t = 2.69 seconds

$$r = \left[(811 \times 2.69)^2 + (6000 + 1400 \times 2.69)^2 - 2 \times 811 \times 2.69 \times .707(6000 + 1400 \times 2.69)\right]^{1/2}$$

= 83[illegible]0 feet

Solution of Line (6) -- sin α'

From NRL 807, Formula (8) page 5:

$$\sin \alpha' = \frac{U \sin \alpha}{V_s}$$

For r_f = 1500 feet, $\sin \alpha' = \frac{4069 \times .707}{3542} = .81219$

For r_f = 3000 feet, $\sin \alpha' = \frac{3733 \times .707}{3211} = .82186$

For r_f = 4500 feet, $\sin \alpha' = \frac{3381 \times .707}{2865} = .83431$

For r_f = 6000 feet, $\sin \alpha' = \frac{3005 \times .707}{2499} = .85014$

Solution of Line (7) -- sin λ

From NRL 807, formula (9) page 5:

$$\sin \lambda = \frac{V_t \, t \sin \alpha}{r}$$

For r_f = 1500 feet, $\sin \lambda = \frac{811 \times .535 \times .707}{1966} = .15603$

For r_f = 3000 feet, $\sin \lambda = \frac{811 \times 1.13 \times .707}{3987} = .16251$

For r_f = 4500 feet, $\sin \lambda = \frac{811 \times 1.82 \times .707}{6095} = .17121$

For r_f = 6000 feet, $\sin \lambda = \frac{811 \times 2.69 \times .707}{8370} = .18428$

Solution of Line (8) -- TNT Equivalent

$\tau = 1.5$ .40 lb MOX

From BRL 807, formula (11) page 6:

$$\text{TNT Equiv (lb)} = \tau WR \left(1 + \frac{V_s^2}{8 \times 10^7 \, \tau R}\right)$$

For r_f = 1500 feet

$$\text{TNT Equiv} = .60 \left(1 + \frac{3542^2}{3.55 \times 10^7}\right) = .810 \text{ lb}$$

For r_f = 3000 feet

$$\text{TNT Equiv} = .60 \left(1 + \frac{3211^2}{3.55 \times 10^7}\right) = .774 \text{ lb}$$

For r_f = 4500 feet

$$\text{TNT Equiv} = .60 \left(1 + \frac{2865^2}{3.55 \times 10^7}\right) = .738 \text{ lb}$$

For r_f = 6000 feet

$$\text{TNT Equiv} = .60 \left(1 + \frac{2499^2}{3.55 \times 10^7}\right) = .708 \text{ lb}$$

Solution of Line (10) -- A_v

From BRL 807, Formula (10) page 6:

$$A_v = A_v' \, (\sqrt{2} \sin \alpha')$$

For r_f = 1500 feet $A_v = 290 \times 1.414 \times .8122 = 333.0$ sq ft

For r_f = 3000 feet $A_v = 282.5 \times 1.414 \times .8219 = 328.35$ sq ft

For r_f = 4500 feet $A_v = 275.0 \times 1.414 \times .8343 = 324.42$ sq ft

For r_f = 6000 feet $A_v = 267.5 \times 1.414 \times .8501 = 321.5$ sq ft

Solution of Line (11) -- C_1

From BRL 807, page 8:

$$C_1 = \left(\frac{50}{r^2}\right) \left\{t(U - V_f)(r - r_f \cos \lambda) - r_f(2r - r_f \cos \lambda)\right\}$$

For r_f = 1500 feet

$$C_1 = \left(\frac{50}{1966^2}\right)\left\{.535(4069 - 1400)(1966 - 1500 \times .9877) - 1500(2 \times 1966 - 1500 \times .9877)\right\}$$

$= -38.59$

For r_f = 3000 feet

$$C_1 = \left(\frac{50}{3987^2}\right)\left\{1.13(3733-1400)(3987-3000 \times .9867) - 3000(2 \times 3987-3000 \times .9867)\right\}$$

$= -38.80$

For r_f = 4500 feet

$$C_1 = \left(\frac{50}{6095^2}\right)\left\{1.82(3381-1400)(6095-4500 \times .9852) - 4500(2 \times 6095-4500 \times .9852)\right\}$$

$= -38.92$

For r_f = 6000 feet

$$C_1 = \left(\frac{50}{8370^2}\right)\left\{2.69(3005-1400)(8370-6000 \times .983) - 6000(2 \times 8370-6000 \times .983)\right\}$$

$= -38.81$

Solution of Line (12) -- C_2

From BRL 807, page 8:

$$C_2 = \left(\frac{.004\ r_f}{r \sin \lambda}\right) \left\{(U - V_f)t - (r_f - r \cos \lambda)\right\}$$

For r_f = 1500 feet

$$C_2 = \left(\frac{.004 \times 1500}{1966 \times .156}\right) \left\{(4069 - 1400).535 - (1500 - 1966 \times .988)\right\}$$

$$= 36.59$$

For r_f = 3000 feet

$$C_2 = \left(\frac{.004 \times 3000}{3987 \times .1625}\right) \left\{(3733 - 1400)1.13 - (3000 - 3987 \times .987)\right\}$$

$$= 66.15$$

For r_f = 4500 feet

$$C_2 = \left(\frac{.004 \times 4500}{6095 \times .1712}\right) \left\{(3381 - 1400)1.82 - (4500 - 6095 \times .985)\right\}$$

$$= 88.13$$

For r_f = 6000 feet

$$C_2 = \left(\frac{.004 \times 6000}{8370 \times .1843}\right) \left\{(3005 - 1400)2.69 - (6000 - 8370 \times .983)\right\}$$

$$= 101.83$$

Solution of Line (13) -- C_3

From BRL 807, page 8:

$$C_3 = (.004t \cot \lambda) \left\{(U - V_f)t + \left(\frac{r}{\cos \lambda} - r_f\right)\right\}$$

For r_f = 1500 feet

$$C_3 = (.004 \times .535 \times 6.33)\left\{(4069-1400).535 + \left(\frac{1966}{.988} - 1500\right)\right\}$$

$$= 25.89$$

For r_f = 3000 feet

$$C_3 = (.004 \times 1.13 \times 6.07)\left\{(3733-1400)1.13 + \left(\frac{3987}{.987} - 3000\right)\right\}$$

$$= 100.72$$

For r_f = 4500 feet

$$C_3 = (.004 \times 1.82 \times 5.75)\left\{(3381-1400)1.82 + \left(\frac{6095}{.985} - 4500\right)\right\}$$

$$= 221.78$$

For r_f = 6000 feet

$$C_3 = (.004 \times 2.69 \times 5.33)\left\{(3005-1400)2.69 + \left(\frac{8370}{.983} - 6000\right)\right\}$$

$$= 392.17$$

Solution of Line (14) -- C_4

From BRL 807, page 8:

$$C_4 = \left(\frac{50t}{r}\right)\left\{(U - V_f)t - r_f\right\}$$

For r_f = 1500 feet

$$C_4 = \left(\frac{50 \times .535}{1966}\right)\left\{(4069-1400).535 - 1500\right\}$$

$= -.981$

For r_f = 3000 feet

$$C_4 = \left(\frac{50 \times 1.13}{3987}\right)\left\{(3733 - 1400)1.13 - 3000\right\}$$

$= -5.16$

For r_f = 4500 feet

$$C_4 = \left(\frac{50 \times 1.82}{6095}\right)\left\{(3381 - 1400)1.82 - 4500\right\}$$

$= -13.36$

For r_f = 6000 feet

$$C_4 = \left(\frac{50 \times 2.69}{8370}\right)\left\{(3005 - 1400)2.69 - 6000\right\}$$

$= -27.04$

Solution of Line (15) -- C_5

From BRL 807, page 8:

$$C_5 = (1.01\,U_o - U)\bar{\delta}$$

U_o = 4400 ft/sec; $\bar{\delta}$ = .137 second

For r_f = 1500 feet $C_5 = (1.01 \times 4400 - 4069).137 = 51.37$

For r_f = 3000 feet $C_5 = (1.01 \times 4400 - 3733).137 = 97.41$

For r_f = 4500 feet $C_5 = (1.01 \times 4400 - 3381).137 = 145.63$

For r_f = 6000 feet $C_5 = (1.01 \times 4400 - 3005).137 = 197.14$

Solution of Line (16) -- C_6

From BRL 807, page 8:

$$C_6 = (.002\ P)\left\{\frac{U}{V_t}\csc\alpha - \cot\alpha\right\}$$

For r_f = 1500 feet $C_6 = (.002 \times 2250)\left\{\frac{4069}{811} \times 1.414 - 1\right\} = 27.42$

For r_f = 3000 feet $C_6 = (.002 \times 4580)\left\{\frac{2733}{811} \times 1.414 - 1\right\} = 50.46$

For r_f = 4500 feet $C_6 = (.002 \times 7050)\left\{\frac{3381}{811} \times 1.414 - 1\right\} = 69.02$

For r_f = 6000 feet $C_6 = (.002 \times 9760)\left\{\frac{3005}{811} \times 1.414 - 1\right\} = 82.75$

Solution of Line (17) -- C_7

From BRL 807, page 8:

$$C_7 = .03(U - V_f)$$

For r_f = 1500 feet	$C_7 = .03(4069 - 1400) = 80.07$
For r_f = 3000 feet	$C_7 = .03(3733 - 1400) = 69.99$
For r_f = 4500 feet	$C_7 = .03(3381 - 1400) = 59.43$
For r_f = 6000 feet	$C_7 = .03(3005 - 1400) = 48.15$

Solution of Line (18) -- ΣC_i^2

r_f	1500 Feet	3000 Feet	4500 Feet	6000 Feet
C_1^2	1489.2	1505.4	1514.8	1506.2
C_2^2	1338.8	4375.8	7766.9	10,369.3
C_3^2	670.3	10,144.5	49,186.4	153,797.3
C_4^2	1.0	26.6	178.5	731.2
C_5^2	2638.9	9488.7	21,208.1	38,864.2
C_6^2	751.8	2546.2	4703.8	6847.6
C_7^2	6411.2	4898.6	3531.9	2318.4
ΣC_i^2	13,301.2	32,985.8	88,150.4	214,434.2

Solution of Line (19) -- σ_x^2

From BRL 807, Formula (14) page 9:

$$\sigma_x^2 = \left(\frac{V_t}{V_s} \sin \alpha\right)^2 \sum_{i=1}^{7} C_i^2$$

For r_f = 1500 feet $\quad \sigma_x^2 = \left(\frac{811}{3542} \times .707\right)^2 \times 13,301 = 348.49$

For r_f = 3000 feet $\quad \sigma_x^2 = \left(\frac{811}{3211} \times .707\right)^2 \times 32,896 = 1052.25$

For r_f = 4500 feet $\quad \sigma_x^2 = \left(\frac{811}{2865} \times .707\right)^2 \times 88,150 = 3526.0$

For r_f = 6000 feet $\quad \sigma_x^2 = \left(\frac{811}{2499} \times .707\right)^2 \times 214,434 = 11,300.0$

Solution of Line (20) -- σ_y^2

From BRL 807, Formula (15) page 9:

$$\sigma_y^2 = (.005\ P)^2$$

For r_f = 1500 feet $\quad \sigma_y^2 = (.005 \times 2250)^2 = 126.56$

For r_f = 3000 feet $\quad \sigma_y^2 = (.005 \times 4580)^2 = 524.41$

For r_f = 4500 feet $\quad \sigma_y^2 = (.005 \times 7050)^2 = 1242.56$

For r_f = 6000 feet $\quad \sigma_y^2 = (.005 \times 9760)^2 = 2381.44$

Solution of Line (21) -- σ_H^2

From BRL 807, page 9:

$$\sigma_H^2 = \left(\frac{P}{1000}\right)^2 \left(\frac{V_T}{V_s}\cos\alpha - \frac{U}{V_s}\right) \sigma_1^2$$

$\sigma_1^2 = 18.046$ for CEP = 5.0 mils, $\alpha = 45^\circ$

For r_f = 1500 feet $\sigma_H^2 = 2.250^2\left(\frac{811}{3542} \times .707 - \frac{4069}{3542}\right)^2 \times 18.046 = 88.97$

For r_f = 3000 feet $\sigma_H^2 = 4.580^2\left(\frac{811}{3211} \times .707 - \frac{3733}{3211}\right)^2 \times 18.046 = 366.42$

For r_f = 4500 feet $\sigma_H^2 = 7.05^2\left(\frac{811}{2865} \times .707 - \frac{3381}{2865}\right)^2 \times 18.046 = 860.97$

For r_f = 6000 feet $\sigma_H^2 = 9.76^2\left(\frac{811}{2499} \times .707 - \frac{3005}{2499}\right)^2 \times 18.046 = 1627.93$

Solution of Line(22)-- σ_V^2

From BRL 807, page 9:

$$\sigma_V^2 = \left(\frac{P}{1000}\right)^2 \sigma_1^2$$

For r_f = 1500 feet $\sigma_V^2 = 2.250^2 \times 18.046 = 91.35$

For r_f = 3000 feet $\sigma_V^2 = 4.580^2 \times 18.046 = 378.53$

For r_f = 4500 feet $\sigma_V^2 = 7.050^2 \times 18.046 = 896.89$

For r_f = 6000 feet $\sigma_V^2 = 9.760^2 \times 18.046 = 1719.06$

<u>Solution of Lines (23) and (24) -- a and b</u>

From BRL 807, page 10:

$\pi ab = A_v;$ $\qquad \frac{a}{b} = 8.85\sqrt{2}\sin\alpha'$ $\qquad$ for bomber target

$a = \frac{A_v}{\pi b}$ $\qquad$ Substituting $\frac{A_v}{\pi b}$ for a above

$$b = \sqrt{\frac{A_v}{\pi \times 8.85\sqrt{2}\sin\alpha'}}$$

For r_f = 1500 feet

$$b = \sqrt{\frac{333}{3.14 \times 8.85 \times 1.414 \times .8122}} = 3.23 \text{ feet}$$

$$a = \frac{333}{3.141 \times 3.23} = 32.83 \text{ feet}$$

For r_f = 3000 feet

$$b = \sqrt{\frac{328.35}{3.14 \times 8.85 \times 1.414 \times .8219}} = 3.19 \text{ feet}$$

$$a = \frac{328.35}{3.141 \times 3.19} = 32.83 \text{ feet}$$

For r_f = 4500 feet

$$b = \sqrt{\frac{324.42}{3.14 \times 8.85 \times 1.414 \times .8343}} = 3.14 \text{ feet}$$

$$a = \frac{324.42}{3.141 \times 3.14} = 32.90 \text{ feet}$$

For r_f = 6000 feet

$$b = \sqrt{\frac{321.5}{3.14 \times 8.85 \times 1.414 \times .8501}} = 3.11 \text{ feet}$$

$$a = \frac{321.5}{3.14 \times 3.11} = 32.92 \text{ feet}$$

Solution of Line (25) -- ϵ_H^2

From BRL 807, Formula (16) page 9:

$$\epsilon_H^2 = 2\sigma_H^2 + a^2 + (U - V_f)^2 \left(\frac{V_t}{V_s} \sin \alpha\right)^2 \frac{T^2}{6} \text{ for ripple fire over } T \text{ seconds.}$$

$T = 1.0$ second

For r_f = 1500 feet

$$\epsilon_H^2 = 2 \times 88.97 + 32.83^2 + (4069 - 1400)^2 \left(\frac{811}{3742} \times .707\right)^2 \times \frac{1}{6}$$

$$= 1255.75 + 31,168 = 32,424$$

$$\epsilon_H = 180$$

For r_f = 3000 feet

$$\epsilon_H^2 = 2 \times 366.42 + 32.83^2 + (3733 - 1400)^2 \left(\frac{811}{3211} \times .707\right)^2 \times \frac{1}{6}$$

$$= 1810.65 + 28,996 = 30,807$$

$$\epsilon_H = 175.5$$

For r_f = 4500 feet

$$\epsilon_H^2 = 2 \times 860.97 + 32.90^2 + (3381 - 1400)^2 \left(\frac{811}{2865} \times .707\right)^2 \times \frac{1}{6}$$

$$= 2804 + 26,215 = 29,019$$

$$\epsilon_H = 170.5$$

For r_f = 6000 feet

$$\epsilon_H^2 = 2 \times 1627.95 + 32.92^2 + (3005 - 1400)^2 \left(\frac{811}{2499} \times .707\right)^2 \times \frac{1}{6}$$

$$= 4339 + 22,671 = 27,010$$

$$\epsilon_H = 164.4$$

Solution of Line (26) -- ϵ_v^2

From BRL 807, Formula (17) page 10:

$$\epsilon_v^2 = 2\sigma_v^2 + b^2$$

For r_f = 1500 feet

$$\epsilon_v^2 = 2 \times 91.35 + 3.23^2 = 193.13$$

$$\epsilon_v = 13.9$$

For r_f = 3000 feet

$$\epsilon_v^2 = 2 \times 378.53 + 3.19^2 = 767.23$$

$$\epsilon_v = 27.7$$

For r_f = 4500 feet

$$\epsilon_v^2 = 2 \times 896.89 + 3.14^2 = 1803.68$$

$$\epsilon_v = 42.5$$

For r_f = 6000 feet

$$\epsilon_v^2 = 2 \times 1719.06 + 3.11^2 = 3447.74$$

$$\epsilon_v = 58.7$$

Solution of Lines (27) through (29) -- E_o

From BRL 807, Formula (18) page 10:

$$E_o = \frac{n\,a\,b}{\tau_H\,\tau_V}$$

For r_f = 1500 feet

$$E_{o\,(n=360)} = \frac{360 \times 32.83 \times 3.23}{180 \times 13.9} = 15.258$$

$$E_{o\,(n=180)} = \frac{15.258}{2} = 7.629$$

$$E_{o\,(n=90)} = \frac{15.258}{4} = 3.814$$

For r_f = 3000 feet

$$E_{o\,(n=360)} = \frac{360 \times 32.83 \times 3.19}{175.5 \times 27.7} = 7.756$$

$$E_{o\,(n=180)} = \frac{7.756}{2} = 3.878$$

$$E_{o\,(n=90)} = \frac{7.756}{4} = 1.939$$

For r_f = 4500 feet

$$E_{o\,(n=360)} = \frac{360 \times 32.90 \times 3.14}{170.5 \times 42.5} = 5.132$$

$$E_{o\,(n=180)} = \frac{5.132}{2} = 2.566$$

$$E_{o\,(n=90)} = \frac{5.132}{4} = 1.283$$

For r_f = 6000 feet

$$E_{o(n=360)} = \frac{360 \times 32.92 \times 3.11}{164.4 \times 58.7} = 3.819$$

$$E_{o(n=180)} = \frac{3.819}{2} = 1.910$$

$$E_{o(n=90)} = \frac{3.819}{4} = .955$$

Solution of Lines (30) and (31) -- C_x and C_y

From BRL 807, page 8:

$$C_x = \frac{\tau_H^2}{2\sigma_x^2}; \qquad C_y = \frac{\tau_y^2}{2\sigma_y^2}$$

For r_f = 1500 feet

$$C_x = \frac{32,424}{2 \times 348.5} = 40.68 \qquad C_y = \frac{193.13}{2 \times 126.56} = .763$$

For r_f = 3000 feet

$$C_x = \frac{30,807}{2 \times 1052.25} = 14.64 \qquad C_y = \frac{767.23}{2 \times 524.41} = .732$$

For r_f = 4500 feet

$$C_x = \frac{29,019}{2 \times 3526} = 4.115 \qquad C_y = \frac{1803.68}{2 \times 1242.56} = .726$$

For r_f = 6000 feet

$$C_x = \frac{27,010}{2 \times 11,300} = 1.195 \qquad C_y = \frac{3447.74}{2 \times 2381.44} = .724$$

Solution of Lines (32) through (34) -- P_K

Use supplementary graphs in "Calculations for Kill Probabilities of 30 to 70mm Gun Mechanism" dated 26 October 1953.

Obtain q. $P_K = 1 - q$

For r_f = 1500 feet $C_x = 40.68$ $C_y = .763$

$E_o = 15.258$	(n = 360)	q = .01 Est.	P_K = .99 Est.	*
$E_o = 7.629$	(n = 180)	q = .05 Est.	P_K = .95 Est.	*
$E_o = 3.814$	(n = 90)	q = .09 Est.	P_K = .91 Est.	*

For r_f = 3000 feet $C_x = 14.64$ $C_y = .732$

		C_y =	.70	.75	.732	
$E_o = 7.756$	(n = 360)	q =	.09	.07	.08	P_K = .92
$E_o = 3.878$	(n = 180)	q =	.17	.16	.16	P_K = .84
$E_o = 1.939$	(n = 90)	q =	.30	.29	.29	P_K = .71

For r_f = 4500 feet $C_x = 4.115$ $C_y = .726$

		C_y =	.70	.75	.726	
$E_o = 5.132$	(n = 360)	q =	.15	.14	.14	P_K = .86
$E_o = 2.566$	(n = 180)	q =	.30	.30	.30	P_K = .70
$E_o = 1.283$	(n = 90)	q =	.49	.47	.48	P_K = .52

For r_f = 6000 feet $C_x = 1.195$ $C_y = .724$

		C_y =	.70	.75	.724	
$E_o = 3.819$	(n = 360)	q =	.27	.26	.26	P_K = .74
$E_o = 1.910$	(n = 180)	q =	.43	.43	.43	P_K = .57
$E_o = .955$	(n = 90)	q =	.63	.63	.63	P_K = .37

* These points fall outside of the family of C_x curves, but are compatible with ripple fire P_K at 1500 feet at the bottom of page 6, "Calculations for Kill Probabilities" dated 26 October 1953. They are also comparable with the pursuit case and fair into $P_K = 1.0$ at $r = 0$.

In order to investigate the effect of a higher rate of fire at long range, let us assume T = .50 second instead of 1.0 second as in the calculations.

For r_f = 6000 feet (See page 6-41)

$$\sigma_H^2 = 4339 + (3005 - 1400)^2 \left(\frac{811}{2499} \times .707\right)^2 \times \frac{.50^2}{6}$$

$$= 10,007$$

$$\sigma_H = 100$$

(See page 6-42)

$$\sigma_V = 58.7$$

K_o is inversely proportional to σ_H. Hence (See page 6-43)

$$K_o = 3.819 \times \frac{164.4}{100} = 6.278$$

(See page 6-44)

$$C_x = \frac{10,007}{22,600} = .4428$$

(See page 6-44)

$$C_y = .724$$

From graph: q = .33, P_K = .67

- -

Similarly, assuming a firing time of .8 second, all other conditions remaining the same

$$\sigma_H^2 = 4339 + 14,509 = 18,848$$

$$\sigma_H = 137.3$$

$$K_o = 3.819 \times \frac{164.4}{137.3} = 4.573$$

$$C_x = \frac{18,848}{22,600} = .834$$

$$C_y = .724$$

q = .28 P_K = .72

- -

Similarly, assuming a firing time T = 2.0 seconds:

$$\tau_H^2 = 4339 + 90{,}684 = 95{,}023$$

$$\tau_H = 308.3$$

$$E_o = 3.819 \times \frac{164.4}{308.3} = 2.045$$

$$C_x = \frac{95{,}023}{22{,}600} = 4.204$$

$$C_y = .724$$

$$q = .37 \qquad P_K = .63$$

Hence, it may be seen that for the full complement of ammunition, i.e., 360 rounds, fired at a future range of 6000 feet, 1.0 second of firing time is about the optimum rate in order to obtain the maximum kill probability.

In order to obtain the optimum firing rate for all conditions on a collision course attack, a detailed study would be required. At any rate, it may be seen that a variable firing rate is desirable depending on conditions encountered.

Comparison of Figures 6-2 and 6-4 (immediately following) shows there is little to choose between the pursuit and 45° offset collision courses against a bomber. Apparently, when the kill probabilities are very high, the increase in target area with geometry is relatively unimportant. However, the improved probability of fighter survival would still recommend attacking on the beam aspect, particularly at shorter ranges. At longer range, the probability of fighter survival increases under the specified combat conditions. The simpler fire control problem may therefore be a determining factor in selecting a pursuit attack in that case.

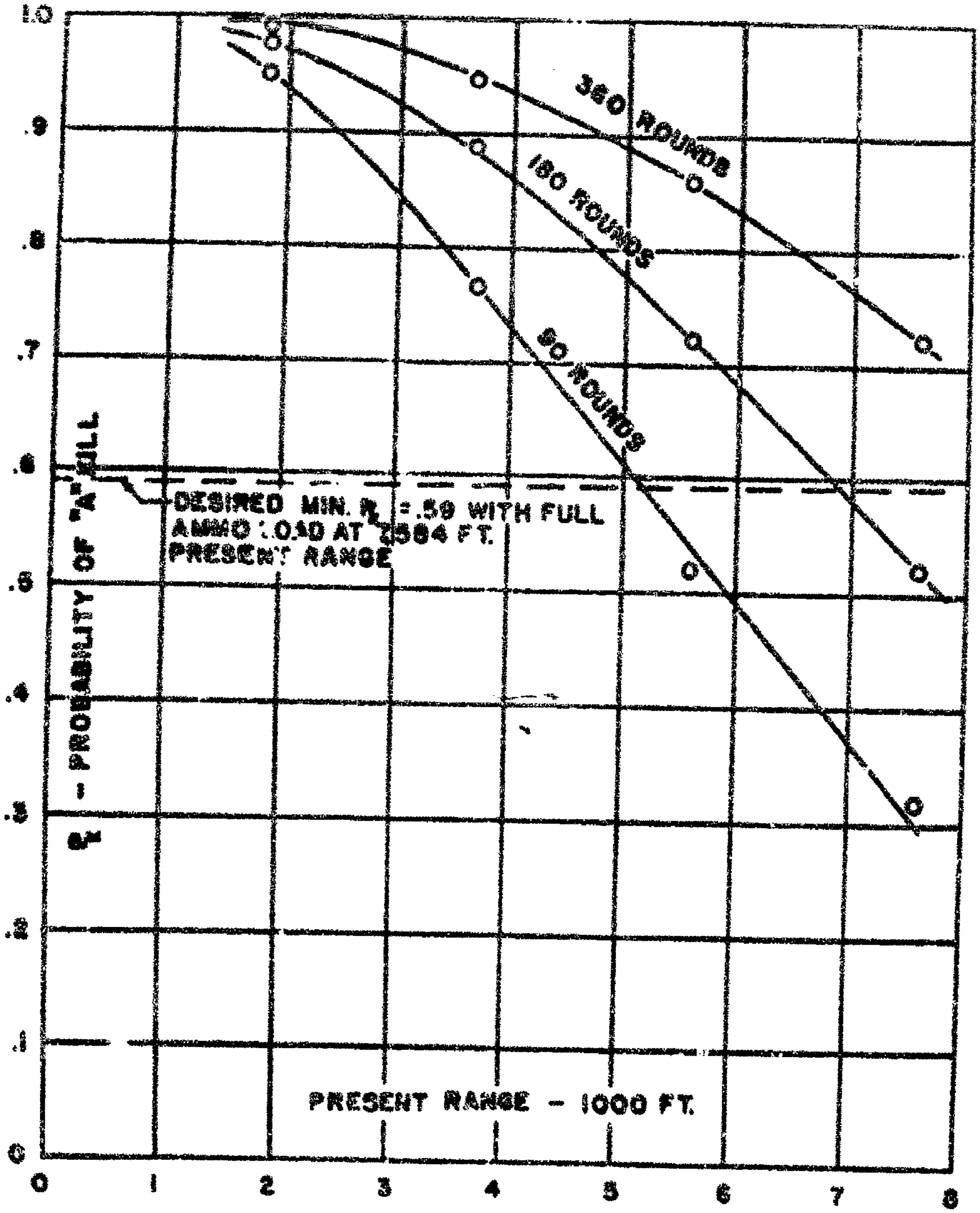

Figure 6-1. Combat No. 1 — Lead Pursuit Course Fighter vs Bomber 20,000 Feet

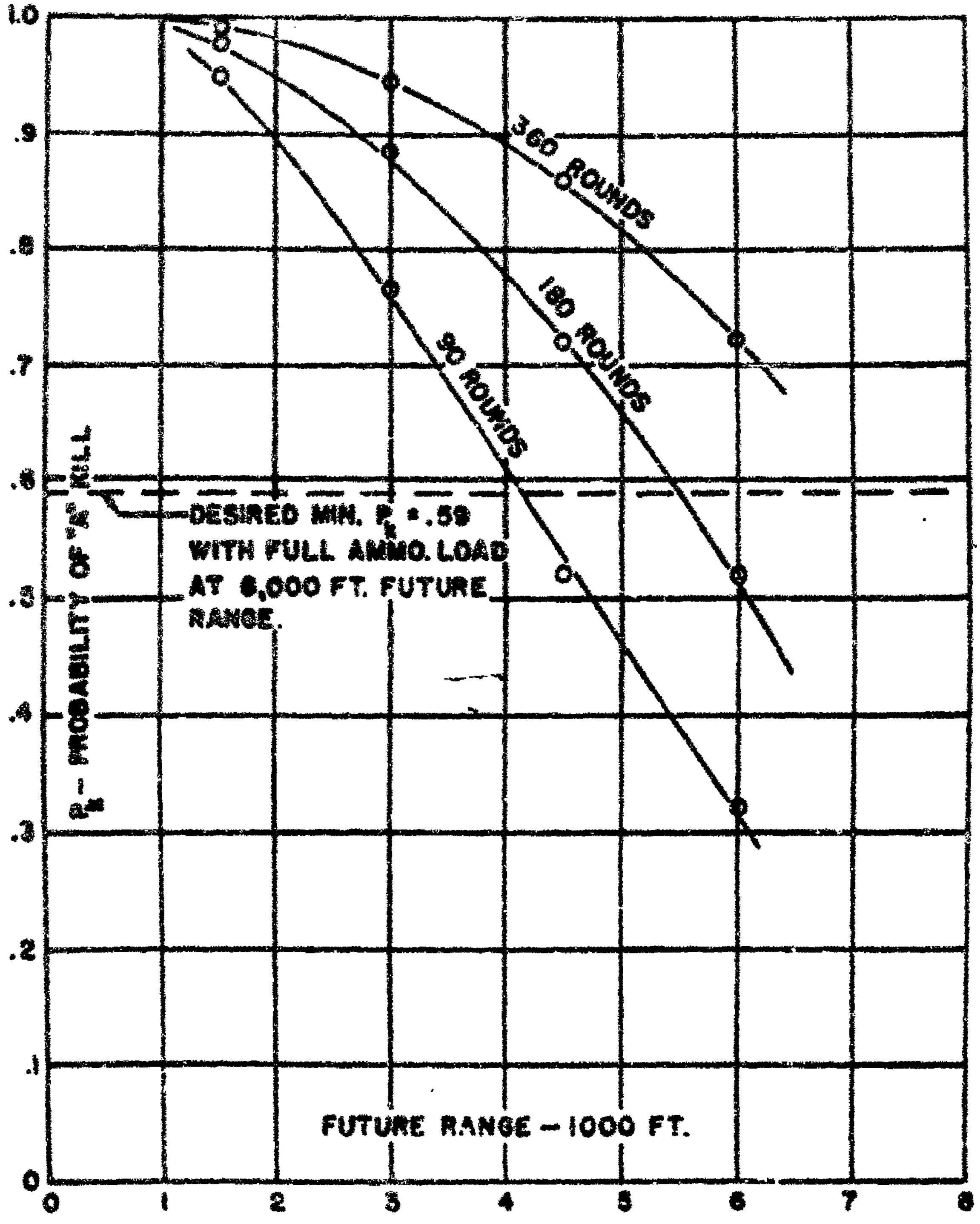

Figure 6-2. Combat No. 1 — Lead Pursuit Course
Fighter vs Bomber 20,000 Feet

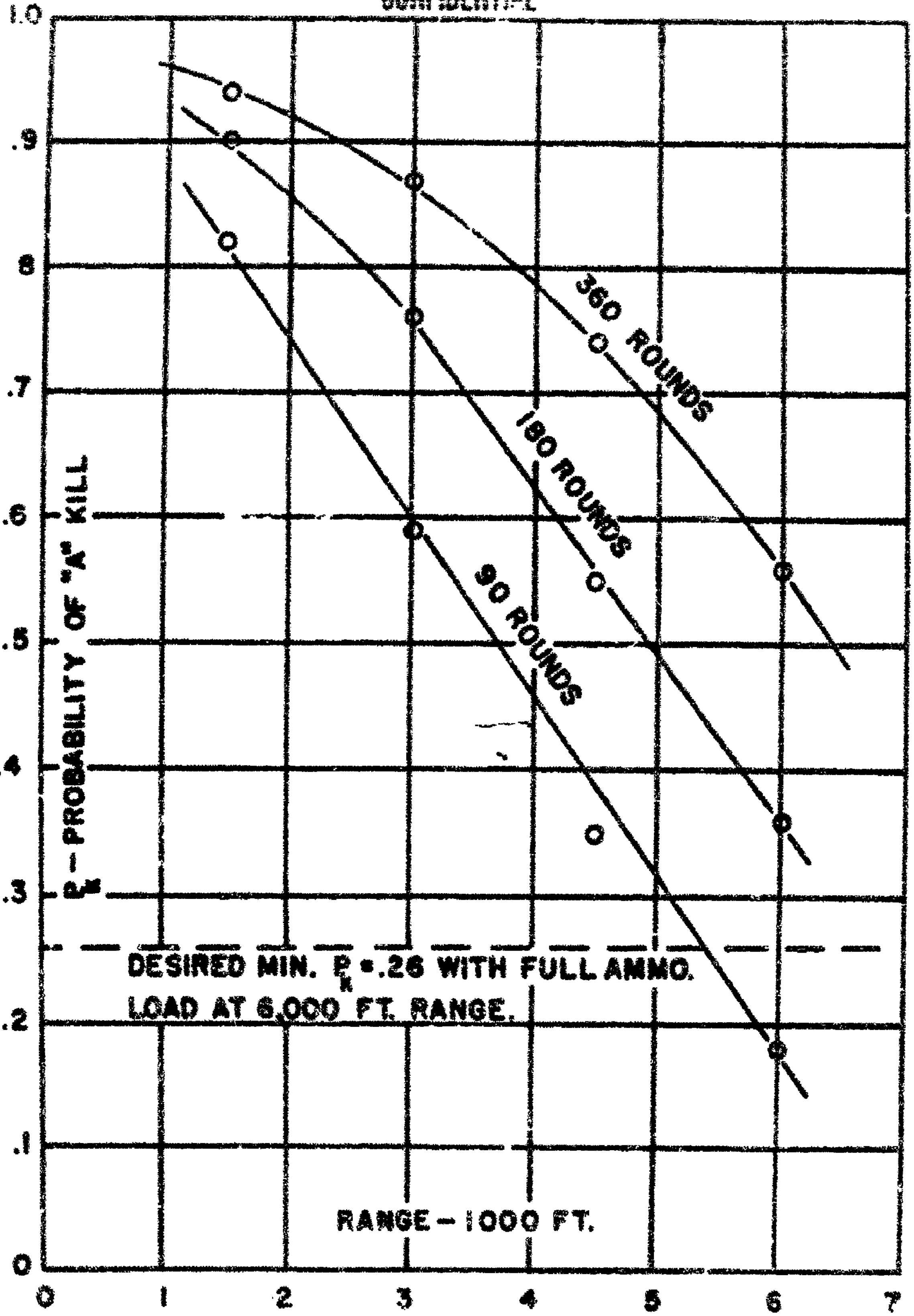

Figure 6-3. Combat No. 2 — Lead Pursuit Course
Fighter vs Fighter 20,000 Feet

[redacted]

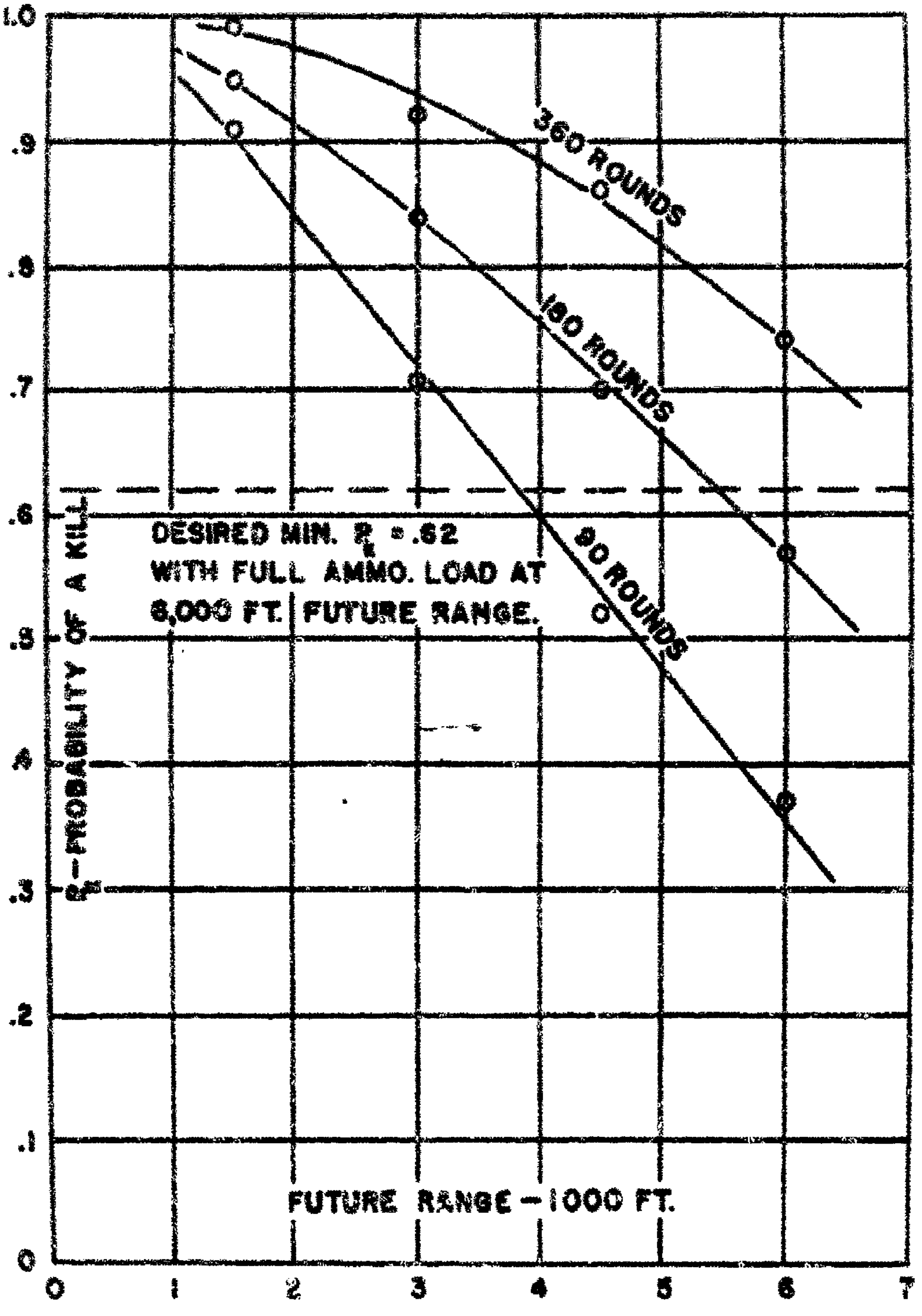

Figure 6-4. Combat No. 3 — Offset Collision Course
Fighter vs Bomber 45° Off Tail 20,000 Feet

[redacted]

7

LOGISTIC CONSIDERATIONS

SECTION 7

LOGISTIC CONSIDERATIONS

The organizing of maintenance and spare parts supply in the field is a major problem in the consideration of combat application of any new weapon. This is especially true of automatic weapons of 20mm and higher caliber. The complexity of automatic mechanisms, per se, causes great complexity of storage, maintenance, and supply.

The number of spares necessary to maintain the open-chamber gun is minimized by comparatively loose tolerances and the elimination of reciprocating parts. It is quite possible that the barrels will need replacement after firing out the full complement of ammunition after one or two missions. Assuming that to be true, the overall expendable weight would be only 12% higher than the ammunition alone. Cost-wise, the percentage of increase (considering the barrels expendable) would be even less. This is believed to be of minor consideration in view of the high kill probabilities achieved.

Ammunition supply is another logistic problem encountered with automatic weapons having high firing rates. This problem is alleviated to some extent by the reduction of storage volume of the special ammunition for the open-chamber gun. This reduction may be as high as 30% over conventional ammunition of similar ballistic performance.

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